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EXISTENCE FOR STOCHASTIC COUPLED SYSTEMS ON NETWORKS WITH TIME-VARYING DELAY DRIVEN BY ROSENBLATT PROCESS WITH DELAY AND POISSON JUMPS

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Abstract. We present some results on the existence and uniqueness of mild solutions for system of semilinear impulsive differential with infinite fractional Brownian motions. Our approach is based on Perov's fixed point theorem and a new version of Schaefer's fixed point theorem in generalized Banach spaces. Also, we investigate the relationship between mild and weak solutions.

1. Introduction

Differential equations with impulses were considered for the first time by Milman and Myshkis [18] and then followed by a period of active research which culminated with the monograph by Halanay and Wexler [13]. Many phenomena and evolution processes in physics, chemical technology, population dynamics, and natural sciences may change state abruptly or be subject to short-term perturbations. These perturbations may be seen as impulses.

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to the basic theory is well developed in the monographs by Benchohra et al [4], Graef *et al* [9], Laskshmikantham *et al.* [2], Samoilenko and Perestyuk [25].

Random differential and integral equations play an important role in characterizing many social, physical, biological and engineering problems; see for instance the monographs by Da Prato and Zabczyk [24], Gard [10], Gikhman and Skorokhod [11], Sobzyk [29] and Tsokos and Padgett [30]. For example, a stochastic model for drug distribution in a biological system was described by Tsokos and Padgett [30] to a closed system with a simplified heat, one organ or capillary bed, and re-circulation of a blood with a constant rate of flow, where the heart is considered as a mixing chamber of constant volume. For the basic theory concerning stochastic differential equations see the monographs by Bharucha-Reid [3], Mao[17], Øksendal, [21], Tsokos and Padgett [30], Sobczyk [29] and Da Prato and Zabczyk [24].

The study of impulsive stochastic differential equations is a new research area. The existence and stability of stochastic of impulsive of differential equations were recently investigated, for example in [8, 9, 14, 15, 16, 23, 22, 31, 32].

This paper is concerned with a system of the following neutral stochastic partial differential equations with delay driven by a Rosenblatt process of the form:

$$\begin{cases} d(x(t) & +g^1(t,x(t-u(t)),y(t-u(t))) = (A_1x(t) \\ & +f^1(t,x(t-r(t)),y(t-r(t)))dt+\sigma^1(t))dZ_1^H(t) \\ & +\displaystyle\int_{\mathcal{Z}} h^1(t,x(t-\rho(t)),y(t-\rho(t)),\kappa)\widetilde{N}(dt,d\kappa),\ t\in[0,b], t\neq t_k, \\ d(y(t) & +g^2(t,x(t-u(t)),y(t-u(t))) = (A_2x(t) \\ & +f^2(t,x(t-r(t)),y(t-r(t)))dt+\sigma^2(t))dZ_2^H(t) \\ & +\displaystyle\int_{\mathcal{Z}} h^2(t,x(t-\rho(t)),y(t-\rho(t)),\kappa)\widetilde{N}(dt,d\kappa),\ t\in[0,b], t\neq t_k, \\ \Delta x(t) & = I_k^1(x(t_k),y(t_k)), \quad k=1,2,\dots,m \\ \Delta y(t) & = \overline{I}_k^2(y(t_k),y(t_k)), \\ x(t) & = \phi_1(t),\ -\tau\leq t\leq 0 \\ y(t) & = \phi_2(t),\ -\tau\leq t\leq 0 \end{cases} \tag{1.1}$$

Here, $x(\cdot), y(\cdot)$ takes the value in the separable Hilbert space X with inner product $\langle\cdot,\cdot\rangle$ induced by the norm $\|\cdot\|$, $A_i : D(A_i) \subset X \longrightarrow X$ is the infinitesimal generator of a strongly continuous semigroup of bounded linear operators $(S_i(t))_{t\geq 0}$ in X for each $i = 1, 2$ and

$f^i, g^i : [0,b]\times X\times X \longrightarrow X$, $Z^H(t)$ is a Rosenblatt process on a real and separable Hilbert space Y with parameter $H \in (\frac{1}{2},1)$, $u(t), r(t) : J \to [0,\tau](\tau > 0)$ are continuous, $\sigma_l^1, \sigma_l^2 : J \to L_Q^0(Y,X)$. Here, $L_Q^0(Y,X)$ denotes the space of all Q_i-Hilbert-Schmidt operators from Y into X, which will be defined in the next section. $I_k, \bar{I}_k \in C(X\times X, X)$ $(k = 1,2,\dots,m)$, $h^1, h^2 : J\times X\times X\times \mathcal{U} \to X$, which will be also defined in the next section (see section 2 below). Moreover, the fixed times t_k satisfies $0 < t_1 < t_2 < \dots < t_m < b$, $y(t_k^-)$ and $y(t_k^+)$ denotes the left and right limits of $y(t)$ at $t = t_k$. As for x we mean the segment solution which is defined in the usual way, that is, if $x(\cdot,\cdot) : [-\tau, b]\times\Omega \to X$, then for any $t \geq 0$. Let $\mathcal{D}_{\mathcal{F}_0}$ be the following space defined by

$$\mathcal{D}_{\mathcal{F}_0} = \big\{\phi_i\colon\ [-\tau,0]\times\Omega\to X \ \text{ is continuous everywhere except for a finite number of points } \phi(t_k^-) \ \text{and}\ \phi(t_k^+) \ \text{with}\ \phi(t_k) = \phi(t_k^-)\big\},$$

endowed with the norm

$$||\phi(t)||_{\mathcal{D}_{\mathcal{F}_0}} = \int_{-\tau}^{0} |\phi(t)|^2 dt.$$

Now, for a given $b > 0$, we define

$$\mathcal{D}_{\mathcal{F}_b} = \big\{x\colon\ [-\tau,b]\times\Omega\to X,\ x_k \in C(J_k, X) \text{ for } k = 1,\dots m,\ \phi_i \in \mathcal{D}_{\mathcal{F}_0},\ \text{and there exist } x(t_k^-) \ \text{and}\ x(t_k^+) \ \text{with}\ x(t_k) = x(t_k^-),\ \ k = 1,\cdots,m,\ \text{and}\ \mathbb{E}(\sup_{t\in[0,b]} \|y(t)\|^2) < \infty\big\},$$

endowed with the norm

$$\|x\|_{\mathcal{D}_{\mathcal{F}_b}} = \mathbb{E}(\sup_{0\le s\le T} \|x(s)\|^2)^{\frac{1}{2}},$$

where x_k denotes the restriction of x to $J_k = (t_{k-1}, t_k]$, $k = 1,2,\cdots,m$, and $J_0 = [-\tau, 0]$.

$$\begin{cases} dz(t) + g_*(t, z(t-u(t)) = A_*z(t) + f(t, z(t-r(t)))dt + \sigma^1(t)dZ^H(t)t_k \\ \qquad\qquad + \displaystyle\int_{\mathcal{Z}} h(t, z(t-\rho(t)),\kappa)\widetilde{N}(dt, d\kappa),\ t\in[0,b], t\neq t_k, \\ \Delta z(t) = I_k^*(z(t_k)), \quad t = t_k \quad k = 1,2,\dots,m \\ z(t) = \phi(t),\ -\tau\le t\le 0 \end{cases} \tag{1.2}$$

where

$$z(t-u(t)) = \begin{bmatrix} x(t-u(t)) \\ y(t-u(t)) \end{bmatrix}, A_* = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix},\ f(t, z(t-r(t)) = \begin{bmatrix} f^1(t, x(t-r(t)), y(t-r(t))) \\ f^2(t, x(t-r(t)), y(t-r(t))) \end{bmatrix}$$

and

$$\sigma(t) = \begin{bmatrix} \sigma^1(t) \\ \sigma^2(t) \end{bmatrix},\ g(t, z(t-r(t)) = \begin{bmatrix} g^1(t, x(t-u(t)), y(t-u(t))) \\ g^2(t, x(t-u(t)), y(t-u(t))) \end{bmatrix}\ \phi(t) = \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix}.$$

and

$$h(t,z(t-\rho(t)),\kappa)=\begin{bmatrix} h^1(t,x(t-\rho(t)),y(t-\rho(t)),\kappa) \\ h^2(t,x(t-\rho(t)),y(t-\rho(t)),\kappa) \end{bmatrix},\ I_k^*(z(t_k))=\begin{bmatrix} I_k^1(x(t_k),y(t_k)) \\ I_k^2(x(t_k),y(t_k)) \end{bmatrix}$$

Some results on the existence of solutions for differential equations with infinite Brownian motion were obtained in [12, 31]. Some existence and uniqueness of mild solutions to neutral stochastic delay functional integro-differential equations perturbed by a fractional Brownian motion can be found in Caraballo and Diop [7].

This paper is organized as follows. In Section 2, we summarize several important working tools on Rosenblatt process, Poisson point processes and we recall some preliminary results about analytic semi-groups and fractional power associated to its generator that will be used to develop our results. In section 3, by Perov's fixed point theorem we consider a sufficient condition for the existence, uniqueness and exponential decay to zero in mean square for mild solutions of equation (1.1). In Section 4, we give an example to illustrate the efficiency of the obtained result.

2. Preliminaries

In this section, we introduce some notations, and recall some definitions, and preliminary facts which are used throughout this paper. Actually we will borrow it from [19, 5]. Although we could simply refer to this paper whenever we need it, we prefer to include this summary in order to make our paper as much self-contained as possible.

Let $(\Omega,\mathcal{F},\mathcal{F}_t,P)$ be a filtered complete probability space satisfying the usual condition, which means that the filtration is a right continuous increasing family and $\mathcal{F}_0$ contains all P-null sets. Suppose $\{p(t),t\geq 0\}$ is a σ-finite stationary $\mathcal{F}_t$-adapted Poisson point process taking values in a measurable space $(\mathcal{U},\mathcal{B}(\mathcal{U}))$. The random measure N_p defined by $N_p((0,t]\times\Lambda):=\sum_{s\in(0,t]}1_\Lambda(p(s))$, for $\Lambda\in\mathcal{B}(\mathcal{U})$ is called the Poisson random measure induced by $p(.)$, thus, we can define the measure $\widetilde{N}.$ by $\widetilde{N}(dt,d\kappa):=N_p(dt,d\kappa)-\nu(dz)dt$, where ν is the characteristic measure of N_p, which is called the compensated Poisson random measure, for a Borel set $\mathcal{Z}\in\mathcal{B}(\mathcal{U}-\{0\})$.

2.1. **Rosenblatt process.** We briefly recall the Rosenblatt process as well as the Wiener integral with respect to it.

Consider $(\xi_n)_{n\in\mathbf{Z}}$ a stationary Gaussian sequence with mean zero and variance 1 such that its correlation function satisfies that $R(n):=E(\xi_0\xi_n)=n^{\frac{2H-2}{k}}L(n)$, with $H\in(\frac{1}{2},1)$ and L

is a slowly varying function at infinity. Let G be a function of Hermite rank k, that is, if G admits the following expansion in Hermite polynomials

$$G(x) = \sum_{j\geq 0} c_j H_j(x), \qquad c_j = \frac{1}{j!} E(GE(\xi_0)H_j(\xi_0)),$$

and

$$H_j(x) = (-1)^j e^{\frac{x^2}{2}} \frac{d^j}{dx^j} e^{\frac{-x^2}{2}}$$

where $H_j(x)$ is the Hermite polynomial of degree j, then $k = \min\{| j |, c_j \neq 0\} \geq 1$, the Non-Central Limit Theorem , $\frac{1}{n^H}\sum_{j=1}^{[nt]} G(\xi_J)$ converges as $n \to \infty$, in the sense of finite dimensional distributions, to the process

$$Z_k^H = c(H,k) \int_{\mathbf{R}^k} \int_0^t \Big(\prod_{j=1}^{k} (s-y_j)_+^{-(\frac{1}{2}+\frac{1-H}{k})} \Big) ds dB(\theta_1) \dots B(\theta_k) \tag{2.3}$$

where the above integral is a Wiener.Ito multiple integral of order k with respect to the standard Brownian motion $(B(\theta))_{\theta\in\mathbf{R}}$ and $c(H,k)$ is a positive normalization constant depending only on H and k. The process $(Z_k^H(t))_{t\geq 0}$ is called as the Hermite process and it is H self-similar in the sense that for any $c > 0$, $(Z_k^H(ct) = c^H Z_k^H(t))$ and it has stationary increments [1].

When $k = 1$ the Hermite process given by (2.3) is the fractional Brownian motion with Hurst parameter $H \in (\frac{1}{2}, 1)$ [34]. If $k = 2$ then the process (2.3) is called as the Rosenblatt process which arises from the Non-Central Limit Theorem (see [35] and references therein). Consider a time interval $[0, T]$ with arbitrary fixed horizon T and let $\{Z^H(t)\ t \in [0, T]\}$ be a one-dimensional Rosenblatt process with parameter $H \in (\frac{1}{2}, 1)$. By Tudor [36], the Rosenblatt process with parameter $H > \frac{1}{2}$ can be written as

$$Z^H(t) = d(H) \int_0^t \int_0^t \int_0^t \Big[\int_{\theta_1\vee\theta_2}^t \frac{\partial K^{H'}}{\partial u}(u,\theta_1) \frac{\partial K^{H'}}{\partial u}(u,\theta_2) du \Big] dB(\theta_1) dB(\theta_2), \tag{2.4}$$

where $K^H(t,s)$ is given by

$$K_H(t,s) = c_H \int_s^t (u-s)^{H-\frac{3}{2}} \left(\frac{u}{s}\right)^{H-\frac{1}{2}} du, \quad t \geq s,$$

where $c_H = \sqrt{\frac{H(2H-1)}{\Gamma(2H-2, H-\frac{1}{2})}}$ and $\Gamma(\cdot,\cdot)$ denotes the Beta function. We put $K^H(t,s) = 0$ if $t \leq s$.

$$\frac{\partial K_H}{\partial t}(t,s) = c_H \left(\frac{t}{s}\right)^{H-\frac{1}{2}} (t-s)^{H-\frac{3}{2}}.$$

where $(B(t), t \in [0,T])$ is a Brownian motion, $H' = \frac{H+1}{2}$ and $d(H) = \frac{1}{H+1}\sqrt{\frac{H}{2(2H-1)}}$ is a normalizing constant. The covariance of the Rosenblatt process $\{Z^H(t), t \in [0,T]\}$ satisfies that $R_H(t,s) = E[Z^H(t))Z^H(s)]$

$$R_H(t,s) = \frac{1}{2}(|t|^{2H} + |s|^{2H} - |t-s|^{2H}) \quad t,s \in [0,T],$$

One note that

$$Z^H(t) = \int_0^T \int_0^T I(\chi_{[0,t)})(\theta_1,\theta_2)dB(\theta_1)dB(\theta_2)$$

where the operator I is defined on the set of functions $f : [0,T] \to \mathbf{R}$, which takes its values in the set of functions $G : [0,T]^2 \to \mathbf{R}^2$ and is given by

$$I(f)(\theta_1,\theta_2) = d(H)\int_{\theta_1 \vee \theta_2}^T f(u)\frac{\partial K^{H'}}{\partial u}(u,\theta_1)\frac{\partial K^{H'}}{\partial u}(u,\theta_2)du$$

Let f be an element of the set $\mathcal{E}$ of step functions on $[0,T]$ of the form

$$f = \sum_{i=1}^{n-1} a_i \chi_{(t_i,t_{i+1}]}, \;\; t_i \in [0,T]$$

Then, it is natural to define its Wiener integral with respect to Z^Has

$$\int_0^T f(u)Z^H(u) := \sum_{i=1}^{n-1} a_i(Z^H(t_{i+1}) - Z^H(t_i)) = \int_0^T \int_0^T I(f)(\theta_1,\theta_2)dB(\theta_1)dB(\theta_2)$$

Let $\mathcal{H}$ be the set of functions f such that

$$\mathcal{H} = \Big\{ f : [0,T] \to \mathbf{R} : \quad \|f\|^2_{\mathcal{H}} = 2\int_0^T \int_0^T (I(f)(\theta_1,\theta_2))^2 d(\theta_1)d(\theta_2) < \infty \Big\}$$

It follows that (see [36])

$$\|f\|^2_{\mathcal{H}} = H(2H-1)\int_0^T \int_0^T f(u)f(v)|u-v|^{2H-2}dudv.$$

It is shown in [1] that the mapping

$$f \to \int_0^T f(u)dZ^H(u)$$

defines an isometry from $\mathcal{E}$ to $L^2(\Omega)$ and it can be extended continuously to an isometry from $\mathcal{H}$ to $L^2(\Omega)$ because $\mathcal{E}$ is dense in $\mathcal{H}$. We call this extension as the Wiener integral of $f \in \mathcal{H}$ with respect to Z^H.

We refer to [36] for the proof of the fact that K.H is an isometry between H and $L^2([0,T])$.

It follows from [36] that H contains not only functions but its elements could be also distributions. In order to obtain a space of functions contained in H, we consider the linear space $|\mathcal{H}|$ generated by the measurable functions f such that

$$\|f\|^2_{|\mathcal{H}|} = H(2H-1)\int_0^T\int_0^T |f(u)||f(v)||u-v|^{2H-2}dudv$$

where $\alpha_H = H(2H-1)$. The space $|H|$ is a Banach space with the norm $\|f\|_{|\mathcal{H}|}$ and we have the following inclusions (see[36]).

As a consequence, we have

$$L^2([0,T]) \subset L^{\frac{1}{H}}([0,T]) \subset |\mathcal{H}| \subset \mathcal{H}$$

For any $f \in L^2([0,T])$, we have

$$\|f\|^2_{|\mathcal{H}|} = 2HT^{2H-1}\int_0^T |f(s)|^2 ds$$

and

$$\|f\|^2_{|\mathcal{H}|} \le C(H)\|f\|^2_{L^{\frac{1}{H}}([0,T])}$$

for some constant $C(H) > 0$. For simplicity throughout this paper we let $C(H) > 0$ stand for a positive constant depending only on H and its value may be different in different appearances.

Consider the linear operator K_H^* from $\mathcal{E}$ to $L^2([0,T])$ defined by

$$(K_H^* f)(\theta_1,\theta_2) = \int_{\theta_1\vee\theta_2}^T f(t)\frac{\partial\mathcal{K}}{\partial t}(t,\theta_1,\theta_2)dt,$$

where $\mathcal{K}$ is the kernel of Rosenblatt process in representation (2.4)

$$\mathcal{K}(t,\theta_1,\theta_2) = \chi_{[0,t]}(\theta_1)\chi_{[0,t]}(\theta_2)\int_{\theta_1\vee\theta_2}^T \frac{\partial K^{H'}}{\partial u}(u,\theta_1)\frac{\partial K^{H'}}{\partial u}(u,\theta_2)du$$

Notice that $(K_H^*\chi_{[0,t]})(\theta_1,\theta_2) = \mathcal{K}(t,\theta_1,\theta_2)\chi_{[0,t]}(\theta_1)\chi_{[0,t]}(\theta_2)$. The operator K_H^* is an isometry between $\mathcal{E}$ to $L^2([0,T])$, which could be extended to the Hilbert space $\mathcal{H}$. In fact, for any $s,t \in [0,T]$ we have

$$\begin{aligned}
\langle K_H^*\chi_{[0,t]}, K_H^*\chi_{[0,s]}\rangle_{L^2([0,T])} &= \langle \mathcal{K}(t,.,.)\chi_{[0,t]}, \mathcal{K}(t,.,.)\chi_{[0,s]}\rangle_{L^2([0,T])} \\
&= \int_0^{t\wedge s}\int_0^{t\wedge s} \mathcal{K}(t,\theta_1,\theta_2)\mathcal{K}(s,\theta_1,\theta_2)d\theta_1 d\theta_2 \\
&= H(2H-1)\int_0^t\int_0^s |u-v|^{2H-2}dudv \\
&= \langle \chi_{[0,t]}, \chi_{[0,s]}\rangle_{\mathcal{H}}.
\end{aligned}$$

Moreover, for $f \in \mathcal{H}$, we have

$$Z^H(f) = \int_0^T \int_0^T K_H^*(f)(\theta_1, \theta_2) dB(\theta_1) dB(\theta_2)$$

Let $\{z_n(t)\}_{n \in \mathbf{N}}$ be a sequence of two-sided one dimensional Rosenblatt process mutually independent on $(\Omega, \mathcal{F}, P)$. We consider a K-valued stochastic process $Z_Q(t)$ given by the following series:

$$Z_Q(t) = \sum_{n=1}^{\infty} z_n(t) Q^{1/2} e_n, \quad t \geq 0$$

Moreover, if Q is a non-negative self-adjoint trace class operator, then this series converges in the space K, that is, it holds that $Z_Q(t) \in L^2(\Omega, K)$. Then, we say that the above $Z_Q(t)$ is a K-valued $Q-$ Rosenblatt process with covariance operator Q. For example, if $\{\sigma_n\}_{n \in \mathbf{N}}$ is a bounded sequence of non-negative real numbers such that $Qe_n = \sigma_n e_n$, assuming that Q is a nuclear operator in K, then the stochastic process

$$Z_Q(t) = \sum_{n=1}^{\infty} z_n(t) Q^{\frac{1}{2}} e_n = \sum_{n=1}^{\infty} z_n(t) \sqrt{\sigma_n} e_n, \ \ t \geq 0$$

is well-defined as a X-valued $Q-$ Rosenblatt process.

Definition 2.1. *Let $\phi : [0, T] \to L_Q^0(Y, X)$ such that $\sum_{n=1}^{\infty} \|K_H^*(\phi Q^{\frac{1}{2}} e_n)\|_{L^2([0,T],X)} < \infty$. Then, its stochastic integral with respect to the Rosenblatt process $Z_Q(t)$ is defined, for $t \geq 0$, as follows:*

$$\int_0^t \phi(s) dZ_Q(s) := \sum_{n=1}^{\infty} \int_0^t \phi(s) Q^{1/2} e_n dz_n(s) = \int_0^t K_H^*(\phi Q^{\frac{1}{2}} e_n)(\theta_1, \theta_2) dB(\theta_1) dB(\theta_2) \quad (2.5)$$

Now, we end this subsection by stating the following result which is fundamental to prove our result.

Lemma 2.1. [6] *For any $\phi : [0, T] \to L_Q^0(Y, X)$ such that $\sum_{n=1}^{\infty} \|\phi Q^{\frac{1}{2}} e_n\|_{L^{\frac{1}{H}}([0,T],X)}$ holds, and for any $\alpha, \beta \in [0, T]$ with $\alpha > \beta$,*

$$E \left\| \int_\alpha^\beta \phi(s) dZ_Q(s) \right\|^2 \leq c_H H(2H-1)(\alpha - \beta)^{2H-1} \sum_{n=1}^{\infty} \int_\alpha^\beta \left\| \phi(s) Q^{1/2} e_n \right\|^2 ds. \quad (2.6)$$

where $c = c(H)$. If, in addition,

$$\sum_{n=1}^{\infty} \|\phi Q^{1/2} e_n\| \text{ is uniformly convergent for } t \in [0, T]$$

then

$$E \left\| \int_\alpha^\beta \phi(s) dB_l^H(s) \right\|^2 \leq c_H H(2H-1)(\alpha - \beta)^{2H-1} \int_\alpha^\beta \|\phi(s)\|_{L_Q^0}^2 ds. \quad (2.7)$$

3. Fixed point results

The classical Banach contraction principle was extended for contractive maps on spaces endowed with vector-valued metric space by Perov in 1964 [27], Precup [26]. Let us recall now some useful definitions and results.

Definition 3.1. *A square matrix of real numbers is said to be convergent to zero if and only if its spectral radius $\rho(M)$ is strictly less than 1. In other words, this means that all the eigenvalues of M are in the open unit disc. (i.e.$|\lambda| < 1$, for every $\lambda \in \mathbb{C}$ with $det(M-\lambda I) = 0$, where I denote the unit matrix of $\mathcal{M}_{n\times n}(\mathbb{R})$).*

Definition 3.2. *We say that a non-singular matrix $A = (a_{ij})_{1\leq i,j\leq n} \in \mathcal{M}_{n\times n}(\mathbb{R})$ has the absolute value property if*

$$A^{-1}|A| \leq I,$$

where

$$|A| = (|a_{ij}|)_{1\leq i,j\leq n} \in \mathcal{M}_{n\times n}(\mathbb{R}_+).$$

Lemma 3.1. [20] *Let M be a square matrix of nonnegative numbers. The following assertions are equivalent:*

(i) *M is convergent towards zero;*

(ii) *the matrix $I - M$ is non-singular and*

$$(I - M)^{-1} = I + M + M^2 + \ldots + M^k + \ldots;$$

(iii) $\|\lambda\| < 1$ *for every $\lambda \in \mathbb{C}$ with $det(M - \lambda I) = 0$*

(iv) *$(I - M)$ is non-singular and $(I - M)^{-1}$ has nonnegative elements;*

Some examples of matrices convergent to zero are the following:

1) $A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, where $a, b \in \mathbb{R}_+$ and $\max(a, b) < 1$

2) $A = \begin{pmatrix} a & -c \\ 0 & b \end{pmatrix}$, where $a, b, c \in \mathbb{R}_+$ and $a + b < 1,\ c < 1$

3) $A = \begin{pmatrix} a & -a \\ b & -b \end{pmatrix}$, where $a, b, c \in \mathbb{R}_+$ and $|a - b| < 1,\ a > 1, b > 0$.

For other examples and considerations on matrices which converge to zero, see Precup [26], Rus [40], and Turinici [39].

We can recall now a fixed point theorem in a complete generalized metric space.

Theorem 3.1. [27]*Let (X,d) be a complete generalized metric space with $d : X \times X \longrightarrow \mathbb{R}^n$ and let $N : X \longrightarrow X$ be such that*

$$d(N(x), N(y)) \leq Md(x, y)$$

for all $x, y \in X$ and some square matrix M of nonnegative numbers. If the matrix M is convergent to zero, that is $M^k \longrightarrow 0$ as $k \longrightarrow \infty$, then N has a unique fixed point $x_* \in X$

$$d(N^k(x_0), x_*) \leq M^k(I - M)^{-1}d(N(x_0), x_0)$$

for every $x_0 \in X$ and $k \geq 1$.

We suppose that $0 \in \rho(A_i)$ (the resolvent set of A_i, for each $i = 1, 2$).that the semigroup $S_i(t)$ is uniformly bounded, that is to say, $\|S_i(t)\| \leq \bar{M}_1$, for some constant $\bar{M}_1 \geq 1$ and for every $t \geq 0$. For $0 < \alpha \leq 1$,it is possible to define the fractional power operator $(-A_i)^\alpha$ as a closed linear operator on its domain $D((-A_i)^\alpha)$ with inverse $(-A_i)^{-\alpha}$. Furthermore, the sub-space $D((-A_i)^\alpha)$ is dense in X. We denote by X_α the Banach space $D((-A_i)^\alpha)$ endowed with the norm $\|x\|_\alpha = \|(-A_i)^\alpha x\|$ for $x \in D((-A_i)^\alpha)$ defines a norm on $D((-A_i)^\alpha)$, which is equivalent to the graph norm of $(-A_i)^\alpha$, we represent X_α the space $D((-A_i)^\alpha)$ with the norm $\| \cdot \|_\alpha$. then the following properties are well known (cf. Pazy. ([37]), p. 74).

Lemma 3.2. **(A):** *If $0 < \beta < \alpha \leq 1$, then $X_\alpha \subset X_\beta$ and the embedding is compact whenever the resolvent operator of A_i is compact.*

(B): *For each $0 < \alpha \leq 1$, there exists a positive constant C_α such that $\|(-A_i)^\alpha S_i(t)\| \leq \frac{C_\alpha}{t^\alpha} e^{-\lambda t}$, $t > 0$, $\lambda > 0$.*

We are now in a position to state and prove our local existence result for the problem (1.1). First we will list the following hypotheses which will be imposed in our main theorem.

- (H1) A_i is the infinitesimal generator of an analytic semigroup, $S_i(t)$ of bounded linear operators on X. Further, to avoid unnecessary notations, we suppose that $0 \in \rho(A_i)$, and that, see Lemma 3.2, and there exists a constant M such that $\{\|S_i(t)\|^2 \leq M\}$ for all $t \geq 0$

$$\|(-A_i)^{1-\beta} S_i(t)\| \leq C_{1-\beta} t^{\beta - 1}$$

  for some constants $M, C_{1-\beta}$ and every $t \in [0, b]$.
- (H2)

(i): There exist constants $0 < \beta < 1$, $L_{g_{i1}} \geq 0$ and g^i is X_β-valued, $(-A_i)^\beta g^i$ is continuous, and

$$\|(-A_i)^\beta g^i(t, y_1, y_2)\|^2 \leq L_{g_{i1}}(1 + \|y_1\|^2 + \|y_2\|^2),\ t \in J,\ y_1, y_2 \in X$$

(ii): There exist constants $0 < \beta < 1$, $L_{g_i}, L_{\bar{g}_i} \geq 0$, and

$$\|(-A_i)^\beta g^i(t, x, y) - (-A_i)^\beta g^i(t, \bar{x}, \bar{y})\| \leq L_{g_i}\|x - \bar{x}\| + L_{\bar{g}_i}\|y - \bar{y}\|,\ t \in J,$$

$x, y, \bar{x}, \bar{y} \in X$

- (H3) The map $f^i : [0, \infty) \times X \times X \to X$ satisfies the following condition: for all $t \geq 0$, $x, y, \bar{x}, \bar{y} \in X$ that is, there exist positive constants $L_{f_i}, L_{\bar{f}_i}$ and $L_{f_{i1}}$,$i = 1, 2$ such that,

$$\|f^i(t, x, y) - f^i(t, \bar{x}, \bar{y})\| \leq L_{f_i}\|x - \bar{x}\| + L_{\bar{f}_i}\|y - \bar{y}\|,$$

and

$$\|f^i(t, x, y)\|^2 \leq L_{f_{i1}}(1 + \|x\|^2 + \|y\|^2),$$

- (H4) There existe a constant $c_i, \bar{c}_i$ for each $i = 1, 2$ such that

$$\|I^i_k(x, y) - I^i_k(\bar{x}, \bar{y})\| \leq c_i\|x - \bar{x}\| + \bar{c}_i\|y - \bar{y}\|,$$

for all $x, \bar{x}$, $y, \bar{y} \in X$ and $t \in J$.

- (H5)The function $\sigma^i : J \longrightarrow L^0_{Q_i}(Y, X)$ satisfies

$$\int_0^b \|\sigma^i(s)\|^2_{L^0_{Q_i}}\, ds < \infty.$$

- (H6) There exists a positive constant $L_{h_{i1}}, L_{\bar{h}_{i1}}$,$i = 1, 2$ such that,

$$\int_{\mathcal{Z}} \|h^i(s, x, y, \kappa) - h^i(s, \bar{x}, \bar{y}, \kappa)\|^2 \nu(d\kappa) \leq L_{h_{i1}}\|x - \bar{x}\|^2 + L_{\bar{h}_{i1}}\|y - \bar{y}\|^2$$

and

$$\int_{\mathcal{Z}} \|h^1(s, x, y, \kappa)\|^2 \nu(d\kappa) \leq L_{h_{i1}}(1 + \|x\|^2 + \|y\|^2)$$

for all $x, \bar{x}$, $y, \bar{y} \in X$ and $t \in J$.

Now, we first define the concept of mild solution to our problem.

Definition 3.3. *Aa $X-$ valued stochastic process $u = (x, y) \in \mathcal{D}_{\mathcal{F}_b} \times \mathcal{D}_{\mathcal{F}_b}$ is called a mild solution of the problem* (1.1) *with respect to the probability space* $(\Omega, \mathcal{F}, \mathbb{P})$*,if:*

1) *$u(t)$ is $\mathcal{F}_t$-adapted for all $t \in J_k = (t_k, t_{k+1}]$ $\quad k = 1, 2, \dots, m$;*
2) *$u(t)$ is right continuous and has limit on the left almost surely;*

3) *$u(t)$ satisfies for all $t \in [-\tau, b]$ and almost surely that,*

$$\begin{cases} x(t) &= S_1(t)(\phi_1(0) + g^1(0, \phi_1(-u(0)), \phi_2(-u(0))) - g^1(t, x(t-u(t)), y(t-u(t))) \\ &- \int_0^t A_1 S_1(t-s) g^1(s, x(s-u(s)), y(s-u(s)))ds \\ &+ \int_0^t S_1(t-s) f^1(s, x(s-r(s)), y(s-r(s)))ds + \int_0^t S(t-s)\sigma^1(s)dZ_{Q_1}(s) \\ &+ \int_0^t S_1(t-s) \int_{\mathcal{Z}} h^1(s, x(s-\rho(s)), y(s-\rho(s)), \kappa)\widetilde{N}(ds, d\kappa) \\ &+ \sum_{0<t<t_k} S_1(t-t_k) I_k^1(x(t_k), y(t_k)) \\ \\ y(t) &= S_2(t)(\phi_2(0) + g^2(0, \phi_1(-u(0)), \phi_2(-u(0))) - g^2(t, x(t-u(t)), y(t-u(t))) \\ &- \int_0^t A_2 S_2(t-s) g^2(s, x(s-u(s)), y(s-u(s)))ds \\ &+ \int_0^t S_2(t-s) f^2(s, x(s-r(s)), y(s-r(s)))ds + \int_0^t S(t-s)\sigma^2(s)dZ_{Q_2}(s) \\ &+ \int_0^t S_2(t-s) \int_{\mathcal{Z}} h^2(s, x(s-\rho(s)), y(s-\rho(s)), \kappa)\widetilde{N}(ds, d\kappa) \\ &+ \sum_{0<t<t_k} S_2(t-t_k) I_k^2(x(t_k), y(t_k)) \end{cases} \tag{3.8}$$

4. Existence and uniqueness of the mild solution

Theorem 4.1. *Suppose that $(H1) - (H6)$ hold and that. Then, problem* (1.1) *possesses a unique mild solution on $[-\tau, b]$.*

Proof. Fix $b > 0$, let $b > 0$, we define

$$\mathcal{D}_{\mathcal{F}_b} = \big\{x\colon [-\tau, b] \times \Omega \to X,\ x_k \in C(J_k, X) \text{ for } k = 1, \dots m,\ \phi_i \in \mathcal{D}_{\mathcal{F}_0}, \text{ and there exist } x(t_k^-) \text{ and } x(t_k^+) \text{ with } x(t_k) = x(t_k^-),\ \ k = 1, \cdots, m, \text{ and } \mathbb{E}(\sup_{t\in[0,b]} \|y(t)\|^2) < \infty\big\},$$

endowed with the norm

$$\|x\|_{\mathcal{D}_{\mathcal{F}_b}} = \mathbb{E}(\sup_{0\le s\le b} \|x(s)\|^2)^{\frac{1}{2}},$$

and

$$S_b(\phi) := \Big\{x \in \mathcal{D}_{\mathcal{F}_T},\ \ x(s) = \phi(s),\ \text{ for }\ s \in [-\tau, 0]\Big\}$$

Then, $S_b(\phi_1)$ is a closed subset of $\mathcal{D}_{\mathcal{F}_b}$ with the norm $\|x\|_{\mathcal{D}_{\mathcal{F}_b}}$. Consider the operator $N : S_b(\phi) \times S_b(\phi) \to S_b(\phi) \times S_b(\phi)$ defined by

$$N(x, y) = (N_1(x, y), N_2(x, y)),\ (x, y) \in S_b(\phi) \times S_b(\phi)$$

where

$$N_1(x(t),y(t)) = \begin{cases} \phi_1(t) \quad t \in [-\tau, 0] \\ \\ S_1(t)(\phi_1(0) + g^1(0,\phi_1(-u(0)),\phi_2(-u(0))) - g^1(t,x(t-u(t)),y(t-u(t))) \\ -\displaystyle\int_0^t A_1 S_1(t-s) g^1(s,x(s-u(s)),y(s-u(s)))ds \\ +\displaystyle\int_0^t S_1(t-s) f^1(s,x(s-r(s)),y(s-r(s)))ds + \int_0^t S_1(t-s)\sigma^1(s)dZ_{Q_1}(s) \\ +\displaystyle\int_0^t S_1((t-s)t-s)\int_{\mathcal{Z}} h^1(s,x(s-\rho(s)),y(s-\rho(s)),\kappa)\widetilde{N}(ds,d\kappa) \\ +\displaystyle\sum_{0<t<t_k} S_1(t-t_k) I_k^1(x(t_k),y(t_k)), \quad \mathbb{P}-a.s, \quad t \in J \end{cases}$$

and

$$N_2(x(t),y(t)) = \begin{cases} \phi_2(t) \quad t \in [-\tau, 0] \\ \\ S_2(t)(\phi_2(0) + g^2(0,\phi_1(-u(0)),\phi_2(-u(0))) - g^2(t,x(t-u(t)),y(t-u(t))) \\ -\displaystyle\int_0^t A_2 S(t-s) g^2(s,x(s-u(s)),y(s-u(s)))ds \\ +\displaystyle\int_0^t S_2(t-s) f^2(s,x(s-r(s)),y(s-r(s)))ds + \int_0^t S_2(t-s)\sigma^2(s)dZ_{Q_2}(s) \\ +\displaystyle\int_0^t S_2(t-s)\int_{\mathcal{Z}} h^2(s,x(s-\rho(s)),y(s-\rho(s)),\kappa)\widetilde{N}(ds,d\kappa) \\ +\displaystyle\sum_{0<t<t_k} S_2(t-t_k) I_k^2(x(t_k),y(t_k)), \quad \mathbb{P}-a.s, \quad t \in J \end{cases}$$

Then it is clear that to prove the existence of mild solutions to equation (1.1) is equivalent to find a fixed point for the operator N.

Now, we aim to prove that the operator N has a fixed point by means of the Perov's fixed point theorem. The proof will be divided into the following two steps.

Step 1. Next we show that $N(x,y)(t) = (N_1(x,y)(t), N_2(x,y)(t))$ is càdlàg process on $S_T(\phi)$. For arbitrary $(x,y) \in S_b(\phi) \times S_b(\phi)$, we will prove that $t \to N(x,y)(t)$ is continuous on the interval $[0,b]$ in the $L^2(\Omega, X)$-sense. Let $0 < t < b$ and $|h|$ be sufficiently small. Then, for any fixed $(x,y) \in S_b(\phi) \times S_b(\phi)$, we have

$$
\begin{aligned}
&\|N_1(x,y)(t+h) - N_1(x,y)(t)\| \\
&\quad \le \|(S_1(t+h) - S_1(t))(\phi_1(0) + g^1(0,\phi_1(-u(0)),\phi_2(-u(0)))\| \\
&+ \|g^1(t+h, x(t+h-u(t+h)), y(t+h-u(t+h))) - g^1(t, x(t-u(t)), y(t-u(t)))\| \\
&\quad + \Big\| \int_0^{t+h} A_1 S_1(t+h-s) g^1(s, x(s-u(s)), y(s-u(s)))ds \\
&- \int_0^t A_1 S_1(t-s) g^1(s, x(s-u(s)), y(s-u(s)))ds \Big\| \\
&\quad + \Big\| \int_0^{t+h} S_1(t+h-s) f^1(s, x(s-r(s)), y(s-r(s)))ds \\
&- \int_0^t S_1(t-s) f^1(s, x(s-r(s)), y(s-r(s)))ds \Big\| \\
&\quad + \Big\| \int_0^{t+h} S_1(t+h-s)\sigma^1(s) dZ_Q(s) - \int_0^t S_1(t-s)\sigma^1(s)dZ_Q(s) \Big\| \\
&+ \Big\| \int_0^{t+h} S_1(t+h-s) \int_{\mathcal{Z}} h^1(s, x(s-\rho(s)), y(s-\rho(s)), \kappa) \widetilde{N}(ds, d\kappa) \\
&\quad - \int_0^t S_1(t-s) \int_{\mathcal{Z}} h^1(s, x(s-\rho(s)), y(s-\rho(s)), \kappa) \widetilde{N}(ds, d\kappa) \Big\| \\
&+ \Big\| \sum_{0<t<t_k+h} S_1(t+h-t_k) I_k^1(x(t_k), y(t_k)) - \sum_{0<t<t_k} S_1(t-t_k) I_k^1(x(t_k), y(t_k)) \Big\| := \sum_{l=1}^{7} J_l^1(h)
\end{aligned}
$$

Put

$$\|N_1(x,y)(t+h) - N_1(x,y)(t)\| = \sum_{l=1}^{7} J_l^1(h) \tag{4.9}$$

Similar computations for N_2 yield

$$\|N_2(x,y)(t+h) - N_2(x,y)(t)\| \le \sum_{l=1}^{7} J_l^2(h) \tag{4.10}$$

We estimate the various terms of the right hand of (4.9) and (4.10) separately.

For the first term, we have

$$\lim_{h\to 0} (S_1(t+h) - S_1(t))(\phi_1(0) + g^i(0, \phi_1(-u(0)), \phi_2(-u(0))) = 0$$

and

$$
\begin{aligned}
J_1^1(h) &= \|(S_1(t+h) - S_1(t))(\phi_1(0) + g^1(0,\phi_1(-u(0)),\phi_2(-u(0)))\| \\
&\le 2M\|\phi_1(0) + g^1(0,\phi_1(-u(0)),\phi_2(-u(0)))\| \in L^2(\Omega)
\end{aligned}
$$

Similarly

$$\begin{aligned}J_1^2(h) &= \|(S_2(t+h) - S_2(t))(\phi_2(0) + g^2(0, \phi_1(-u(0)), \phi_2(-u(0)))\| \\ &\le 2M\|\phi_2(0) + g^2(0, \phi_1(-u(0)), \phi_2(-u(0)))\| \in L^2(\Omega)\end{aligned}$$

Hence, by the Lebesgue dominated theorem, we obtain

$$\lim_{h\to 0} \mathbb{E}\|J_1^i(h)\|^2 = 0, \qquad i = 1, 2$$

By using assumption $(H2)$ and the fact that the operator $(-A)^{-\beta}$ is bounded, we obtain that

$$\begin{aligned}\mathbb{E}|J_2^i(h)|^2 &= \mathbb{E}\Big\|(-A_i)^{-\beta}(-A_i)^{\beta}\Big(g^i(t+h, x(t+h-u(t+h)), y(t+h-u(t+h))) \\ &- g^i(t, x(t-u(t)), y(t-u(t)))\Big)\Big\|^2 \\ &\le \|(-A_i)^{-\beta}\|^2\mathbb{E}\Big\|(-A_i)^{\beta}\Big(g^i(t+h, x(t+h-u(t+h)), y(t+h-u(t+h))) \\ &- g^i(t, x(t-u(t)), y(t-u(t)))\Big)\Big\|^2 \\ &\to 0, \ \ h \to 0.\end{aligned}$$

To estimate $J_3^i(h)$ for each $i = 1, 2$. We consider only the case that $h > 0$ (for $h < 0$ we have the similar estimates hold).

$$\begin{aligned}J_3^i(h) \le & \Big\|\int_0^t A_iS_i(t+h-s)g^i(s, x(s-u(s)), y(s-u(s)))ds \\ &- \int_0^t A_iS_i(t-s)g^i(s, x(s-u(s)), y(s-u(s)))ds\Big\| \\ &+ \Big\|\int_t^{t+h} A_iS_i(t+h-s)g^i(s, x(s-u(s)), y(s-u(s)))ds\Big\| \\ &\le \Big\|\int_0^t (S_i(h) - I)(-A_i)^{1-\beta}S_i(t-s)(-A_i)^{-\beta}g^i(s, x(s-u(s)), y(s-u(s)))ds\Big\| \\ &+ \Big\|\int_0^t (-A_i)^{1-\beta}S_i(t+h-s)(-A_i)^{-\beta}g^i(s, x(s-u(s)), y(s-u(s)))ds\Big\| \\ &:= J_{31}^i(h) + J_{32}^i(h).\end{aligned}$$

For the term $J_{31}^i(h)$. We have

$$\mathbb{E}|J_{31}^i(h)|^2 \le t\mathbb{E}\int_0^t \|(S_i(h) - I)(-A_i)^{1-\beta}S_i(t-s)(-A_i)^{-\beta}g^i(s, x(s-u(s)), y(s-u(s)))\|^2ds$$

and

$$\lim_{h\to 0} \|(S_i(h) - I)(-A_i)^{1-\beta}S_i(t-s)(-A_i)^{-\beta}g^i(s, x(s-u(s)), y(s-u(s)))\| = 0$$

since the strong continuity of $S(t)$. By conditions $(H1)$ and $(H2)$, we have

$$\begin{aligned}J^1_{31}(h) =& \|(S_1(h)-I)(-A_1)^{1-\beta}S_1(t-s)(-A_1)^{-\beta}g^1(s,x(s-u(s)),y(s-u(s)))\|^2\\ \leq& \frac{4M^2M^2_{1-\beta}}{(t-s)^{2-2\beta}}\|(-A_1)^{-\beta}g^1(s,x(s-u(s)),y(s-u(s)))\|^2\\ \leq& L_{g_{11}}\frac{4M^2M^2_{1-\beta}}{(t-s)^{2-2\beta}}(1+\|x\|^2+\|y\|^2).\end{aligned}$$

Similarly

$$J^2_{31}(h) \leq L_{g_{12}}\frac{4M^2M^2_{1-\beta}}{(t-s)^{2-2\beta}}(1+\|x\|^2+\|y\|^2).$$

Hence, by the Lebesgue dominated theorem, we have

$$\lim_{h\to 0}\mathbb{E}|J^i_{31}(h)|^2 = 0.$$

Now, we estimate the term $J_{32}(h)$.

$$\begin{aligned}E|J^1_{32}(h)|^2 \leq& \int_t^{t+h}\Big\|(-A_1)^{1-\beta}S_1(t+h-s)\Big\|^2 ds\times\int_t^{t+h}\Big\|(-A_1)^{-\beta}g^1(s,x(s-u(s)),y(s-u(s)))\Big\|^2 ds\\ \leq& \int_t^{t+h}\frac{M^2_{1-\beta}}{(t+h-s)^{2-2\beta}}ds\times\int_t^{t+h}\Big\|(-A_1)^{-\beta}g^1(s,x(s-u(s)),y(s-u(s)))\Big\|^2 ds\\ \leq& \int_t^{t+h}\frac{M^2_{1-\beta}}{(t+h-s)^{2-2\beta}}ds\times\int_0^b(1+\|x\|^2+\|y\|^2)ds\\ \leq& L_{g_{11}}\frac{M^2_{1-\beta}h^{2\beta-1}}{2\beta-1}\int_0^b(1+\|x\|^2+\|y\|^2)ds\to 0,\quad h\to 0.\end{aligned}$$

Similarly

$$\mathbb{E}|J^2_{32}(h)|^2 \leq L_{g_{12}}\frac{M^2_{1-\beta}h^{2\beta-1}}{2\beta-1}\int_0^b(1+\|x\|^2+\|y\|^2)ds\ \to 0,\quad h\to 0$$

For the term $J^i_4(h)$. We consider also only the case that $h>0$ (for $h<0$ we have the similar estimates hold).

$$\begin{aligned}J^i_4(h) \leq& \Big\|\int_0^t S_i(t+h-s)f^i(s,x(s-r(s)),y(s-r(s)))\\ &- S_i(t-s)f^i(s,x(s-r(s)),y(s-r(s)))ds\Big\|\\ &+\Big\|\int_t^{t+h}S_i(t+h-s)f^i(s,x(s-r(s)),y(s-r(s)))ds\Big\|\\ \leq& \Big\|\int_0^t(S_i(h)-I)S_i(t-s)f^i(s,x(s-r(s)),y(s-r(s)))ds\Big\|\\ &+\Big\|\int_t^{t+h}S_i(t+h-s)f^i(s,x(s-r(s)),y(s-r(s)))ds\Big\|.\end{aligned}$$

By assumption (H3), we have

$$\begin{aligned}
&\mathbb{E}|J_4^i(h)|^2 \\
&\leq t\int_0^t \Big\|(S_i(h)-I)S_i(t-s)f^i(s,x(s-r(s)),y(s-r(s)))\Big\|^2 ds \\
&+ M^2 h\int_t^{t+h} \Big\|f^i(s,x(s-r(s)),y(s-r(s)))\Big\|^2 ds.
\end{aligned}$$

Noting that

$$\lim_{h\to 0}\Big\|(S_i(h)-I)S_i(t-s)f^i(s,x(s-r(s)),y(s-r(s)))\Big\|^2 = 0.$$

By conditions (H1) and (H2), we have

$$\begin{aligned}
\Big\|((S_1(h)-I)S_1(t-s)f^1(s,x(s-r(s)),y(s-r(s)))\Big\|^2 &\leq 4M^4\Big\|f^1(s,x(s-r(s)),y(s-r(s)))\Big\|^2 \\
&\leq 4L_{f_{11}}M^4(1+\|x\|^2+\|y\|^2)
\end{aligned}$$

Similarly for $t\in[0,b]$, we have the estimate

$$\Big\|((S_2(h)-I)S_2(t-s)f^2(s,x(s-r(s)),y(s-r(s)))\Big\|^2 \leq 4L_{f_{12}}M^4(1+\|x\|^2+\|y\|^2).$$

Hence, by the Lebesgue dominated theorem, we have

$$\lim_{h\to 0}\mathbb{E}|J_4^i(h)|^2 = 0.$$

For the term $J_5^i(h)$, see details [38] . By condition (H5), Lemma 2.1 and the Lebesgue dominated theorem , we have

$$\begin{aligned}
&E|J_5^i(h)|^2 \\
&= E\Big\|\int_0^{t+h} S_i(t+h-s)\sigma^i(s)dZ_Q(s) - \int_0^t S_i(t-s)\sigma^i(s)ds\Big\|^2 \\
&\leq E\Big\|\int_0^t (S_i(t+h-s)-S_i(-s)\sigma^i(s)dZ_Q(s)\Big\|^2 + E\Big\|\int_t^{t+h} S_i(t+h-s)\sigma^i(s)dZ_Q(s)\Big\|^2 \\
&\leq C(H)t^{2H-1}\int_0^t \|(S_i(h)-I)S_i(t-s)\sigma^i(s)\|^2 ds + C(H)h^{2H-1}\mathbb{E}\int_t^{t+h}\Big\|S_i(t+h-s)\sigma^i(s)\Big\|^2 ds \\
&\leq C(H)b^{2H-1}M^2\int_0^t \|(S_i(h)-I)\sigma^i(s)\|^2 ds + C(H)h^{2H-1}\mathbb{E}\int_t^{t+h}\Big\|S_i(t+h-s)\sigma^i(s)\Big\|^2 ds \\
&\to 0,\ \ h\to 0.
\end{aligned}$$

Since

$$\lim_{h\to 0}\|(S_i(h)-I)S_i(t-s)\sigma^1(s)\|^2 = 0$$

and

$$\|(S_i(h)-I)\sigma^i(s)\|^2 \le 2M^2\|\sigma^i(s)\|^2 < \infty$$

$$\|(S_i(h)-I)\sigma^i(s)\|^2 \le M^2\|\sigma^i(s)\|^2 < \infty.$$

The condition (H4) assures that

$$\mathbb{E}|J_6^i(h)|^2 \le M^2 \sum_{0<t_k<t} \Big\|(S_i(h)-I)I_k^i(x(t_k),y(t_k))\Big\|^2 + \sum_{t<t_k<t+h} \Big\|S_i(t+h-t_k)I_k^i(x(t_k),y(t_k))\Big\|^2.$$

Noting that

$$\lim_{h\to 0}\Big\|(S_i(h)-I)S_i(t-t_k)I_k^i(x(t_k),y(t_k))\Big\|^2 = 0.$$

By assumption (H6), we have

$$\begin{aligned}
&\mathbb{E}|J_7^i(h)|^2\\
&= 2\mathbb{E}\Big\|\int_0^t (S_i(t+h-s)-S_i(t-s))\int_{\mathcal{Z}} h^i(s,x(s-\rho(s)),y(s-\rho(s)),\kappa)\widetilde{N}(ds,d\kappa)\Big\|^2\\
&+ 2\mathbb{E}\Big\|\int_t^{t+h} S_i(t+h-s)\int_{\mathcal{Z}} h^i(s,x(s-\rho(s)),y(s-\rho(s)),\kappa)\widetilde{N}(ds,d\kappa)\Big\|^2\\
&\le 2M^2\|S_i(h)-I\|^2 E\mathbb{E}\int_0^t\int_{\mathcal{Z}} \|h^i(s,x(s-\rho(s)),y(s-\rho(s)),\kappa)\|^2\nu(d\kappa)ds\\
&\le 2M^2\mathbb{E}\int_t^{t+h}\int_{\mathcal{Z}} \|h^i(s,x(s-\rho(s)),y(s-\rho(s)),\kappa)\|^2\nu(d\kappa)ds,
\end{aligned}$$

and

$$\int_0^t\int_{\mathcal{Z}} \|h^1(s,x(s-\rho(s)),y(s-\rho(s)),\kappa)\|^2\nu(d\kappa)ds \le L_{h_{11}}t(1+\|x\|^2+\|y\|^2) < \infty.$$

Using the strong continuity of $S_i(t)$ and the Lebesgue dominated theorem , we get

$$\lim_{h\to 0}\mathbb{E}|J_7^i(h)|^2 = 0$$

The above arguments show that

$$\|N(x,y)(t+h)-N(\bar{x},\bar{y})(t)\| = \begin{pmatrix}\|N_1(x,y)(t+h)-N_1(\bar{x},\bar{y})(t)\|\\ \|N_2(x,y)(t+h)-N_2(\bar{x},\bar{y})(t)\|\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \text{ as } h\to 0.$$

The above arguments show that $N(x,y)(t)$ is càdlàg process. Then, we conclude that $N(S_b(\phi)\times S_b(\phi)) \subset S_b(\phi)\times S_b(\phi)$

Step 2.Now, we are going to show that $N : S_b(\phi) \times S_b(\phi) \to S_b(\phi) \times S_b(\phi)$ is a contraction mapping. For this end, fixe $x, y \in S_b(\phi) \times S_b(\phi)$, we have

$$
\begin{aligned}
&\|N_1(x,y)(t) - N_1(\bar{x},\bar{y})(t)\|^2 \\
&\leq 5\|(-A_1)^{-\beta}\|^2 \Big\|(-A_1)^{\beta}(g^1(t, x(t-u(t)), y(t-u(t))) - g^1(t, \bar{x}(t-u(t)), \bar{y}(t-u(t))))\Big\|^2 \\
&+ 5\Big\|\int_0^t (-A_1)^{1-\beta} S_1(t-s)(-A_1)^{\beta}(g^1(s, x(s-u(s)), y(s-u(s))) - g^1(s, \bar{x}(s-u(s)), \bar{y}(s-u(s))))ds\Big\|^2 \\
&+ 5\Big\|\int_0^t S_1(t-s)(f^1(s, x(s-r(s)), y(s-r(s))) - f^1(s, \bar{x}(s-r(s)), \bar{y}(s-r(s))))ds\Big\|^2 \\
&+ 5\Big\|\int_0^t S_1(t-s)\int_{\mathcal{Z}} h^1(s, x(s-\rho(s)), y(s-\rho(s)), \kappa) - h^1(s, \bar{x}(s-\rho(s)), \bar{y}(s-\rho(s)), \kappa)\widetilde{N}(ds, d\kappa)\Big\|^2 \\
&+ 5\Big\|\sum_{0<t<t_k} S_1(t-t_k)(I_k^1(x(t_k), y(t_k)) - I_k^1(\bar{x}(t_k), \bar{y}(t_k)))\Big\|^2.
\end{aligned}
$$

From assumption (H_1)-(H_6) and Lemma 2.1, yields that,

$$
\begin{aligned}
&\mathbb{E}\|N_1(x,y)(t) - N_1(\bar{x},\bar{y})(t)\|^2 \\
&\leq 5\|(-A_1)^{-\beta}\|^2 \Big(L_{g_1}^2 E\|x(t-u(t)) - \bar{x}(t-u(t))\|^2 + L_{\bar{g}_1}^2 E\|y(t-u(t)) - \bar{y}(t-u(t))\|^2\Big) \\
&+ 5M_{1-\beta}^2 \frac{t^{2\beta-1}}{2\beta-1}\Big(L_{g_1}^2 \int_0^t E\|x(s-u(s)) - \bar{x}(s-u(s))\|^2 ds + L_{\bar{g}_1}^2 \int_0^t E\|y(s-u(s)) - \bar{y}(s-u(s))\|^2 ds\Big) \\
&+ 5tM^2\Big(L_{f_1}^2 \int_0^t \mathbb{E}\|x(s-u(s)) - \bar{x}(s-u(s))\|^2 ds + L_{\bar{f}_1}^2 \int_0^t \mathbb{E}\|y(s-u(s)) - \bar{y}(s-u(s))\|^2 ds\Big) \\
&+ 5tM^2\Big(L_{h_1}^2 \int_0^t \mathbb{E}\|x(s-u(s)) - \bar{x}(s-u(s))\|^2 ds + L_{\bar{h}_1}^2 \int_0^t \mathbb{E}\|y(s-u(s)) - \bar{y}(s-u(s))\|^2 ds\Big) \\
&+ 5M^2\Big(c_1\mathbb{E}\|x(t) - \bar{x}(t)\|^2 + \bar{c}_1 E\mathbb{E}\|y(t) - \bar{y}(t)\|^2\Big).
\end{aligned}
$$

Then we have

$$
\begin{aligned}
&\mathbb{E}(\sup_{s\in[-\tau,t]}\|N_1(x,y)(s)-N_1(\bar{x},\bar{y})(s)\|^2)\\
&\leq \int_0^t \alpha(s)e^{\tau\widehat{\alpha}(s)}e^{-\tau\widehat{\alpha}(s)}\mathbb{E}(\sup_{s\in[-\tau,t]}\|x(s)-\overline{x}(s)\|^2)ds+5M^2c_1E(\sup_{t\geq 0}\|x(t)-\bar{x}(t)\|^2)\\
&+\int_0^t \alpha(s)e^{\tau\widehat{\alpha}(s)}e^{-\tau\widehat{\alpha}(s)}\mathbb{E}(\sup_{s\in[-\tau,t]}\|y(s)-\overline{y}(s)\|^2)ds+5M^2\bar{c}_1E(\sup_{t\geq 0}\|y(t)-\bar{y}(t)\|^2)\\
&\leq \int_0^t \alpha(s)e^{\tau\widehat{\alpha}(s)}ds\|x-\overline{x}\|_*^2+\int_0^t \alpha(s)e^{\tau\widehat{\alpha}(s)}ds\|y-\overline{y}\|_*^2\\
&+\bar{C}_1\Big(E(\sup_{t\geq 0}\|x(t)-\bar{x}(t)\|^2)+\mathbb{E}(\sup_{t\geq 0}\|y(t)-\bar{y}(t)\|^2)\Big)\\
&\leq \frac{1}{\tau}\int_0^t (e^{\tau\widehat{\alpha}(s)})'ds\|x-\overline{x}\|_*^2+\frac{1}{\tau}\int_0^t (e^{\tau\widehat{\alpha}(s)})'ds\|y-\overline{y}\|_*^2\\
&+e^{\tau\widehat{\alpha}(t)}e^{-\tau\widehat{\alpha}(t)}\bar{C}_1\Big(\mathbb{E}(\sup_{t\geq 0}\|x(t)-\bar{x}(t)\|^2)+E(\sup_{t\geq 0}\|y(t)-\bar{y}(t)\|^2\Big)\\
&\leq (\frac{1}{\tau}+\bar{C}_1)e^{\tau\widehat{\alpha}(t)}\|x-\overline{x}\|_*^2+(\frac{1}{\tau}+\bar{C}_1)e^{\tau\widehat{\alpha}(t)}\|y-\overline{y}\|_*^2,
\end{aligned}
$$

where

$$\bar{C}_1=\max\{4M^2c_1,4M^2\bar{c}_1\},$$

and

$$\gamma_i(t)=5\|(-A_i)^{-\beta}\|^2L_{g_i}^2+5M_{1-\beta}^2\frac{t^{2\beta}}{2\beta-1}L_{g_i}^2+5t^2M^2L_{f_i}^2+5M^2c_i$$

and

$$\bar{\gamma}_i(t)=5\|(-A_i)^{-\beta}\|^2L_{\bar{g}_i}^2+5M_{1-\beta}^2\frac{t^{2\beta}}{2\beta-1}L_{\bar{g}_i}^2+5t^2M^2L_{\bar{f}_i}^2+5M^2\bar{c}_i.$$

Therefore

$$e^{-\tau\widehat{\alpha}(t)}\mathbb{E}\Big(\sup\nolimits_{t\in[-\tau,b]}\|N_1(x(t),y(t))-N_1(\overline{x}(t),\overline{y}(t))\|^2\Big) \quad\leq\quad (\tfrac{1}{\tau}+\bar{C}_1)\Big[\|x-\overline{x}\|_*^2+\|y-\overline{y}\|_*^2\Big],$$

where $\|\cdot\|_*$ is the Bielecki-type norm on $S_b(\phi)$ defined by

$$\|x\|_*^2=\mathbb{E}(\sup_{t\in[0,b]}\|x(t,.)\|^2)e^{-\tau\widehat{\alpha}(t)}$$

where

$$\widehat{\alpha}(t)=\int_0^t\alpha(s)ds,\quad t\in[0,b],$$

and

$$\alpha(s)=\max\{\gamma_i(t),\bar{\gamma}_i(t)\}.$$

Hence

$$\|N_1(x,y)-N_1(\overline{x},\overline{y})\|_*^2 \quad\leq\quad (\tfrac{1}{\tau}+C_1)\|x-\overline{x}\|_*^2+(\tfrac{1}{\tau}+\bar{C}_1)\|y-\overline{y}\|_*^2.$$

Using the fact that for all $a, b \geq 0$ we have $\sqrt{a+b} \leq \sqrt{a} + \sqrt{b}$, we conclude that

$$\|N_1(x,y) - N_1(\overline{x},\overline{y})\|_* \leq \sqrt{\tfrac{1}{\tau} + \bar{C}_1}\|x-\overline{x}\|_* + \sqrt{\tfrac{1}{\tau} + \bar{C}_1}\|y-\overline{y}\|_*.$$

Similar computations for N_1 yield

$$\|N_2(x,y) - N_2(\overline{x},\overline{y})\|_* \leq \sqrt{\tfrac{1}{\tau} + \bar{C}_2}\|x-\overline{x}\|_* + \sqrt{\tfrac{1}{\tau} + \bar{C}_2}\|y-\overline{y}\|_*.$$

where

$$\bar{C}_2 = \max\{4M^2 c_2, 4M^2 \bar{c}_2\}, \ \ \tau^{'} = \max\left\{\frac{\tau}{1+\tau C_1}, \frac{\tau}{1+\tau C_2}\right\}.$$

Thus

$$\begin{aligned} \|N(x,y) - N(\overline{x},\overline{y})\|_* &= \begin{pmatrix} \|N_1((x,y) - N_1(\overline{x},\overline{y})\|_* \\ \|N_2(x,y) - N_2(\overline{x},\overline{y})\|_* \end{pmatrix} \\ &\leq \frac{1}{\sqrt{\tau'}} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \|x-\overline{x}\|_* \\ \|y-\overline{y}\|_* \end{pmatrix}. \end{aligned}$$

Hence

$$\|N(x,y) - N(\overline{x},\overline{y})\|_* \leq \frac{1}{\sqrt{\tau'}} M_{\alpha,\beta} \begin{pmatrix} \|x-\overline{x}\|_* \\ \|y-\overline{y}\|_* \end{pmatrix},$$

for all $(x,y), (\overline{x},\overline{y}) \in S_b(\phi) \times S_b(\phi)$, where

$$M_{\alpha,\beta} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

If we choose a suitable $\sqrt{\tau'} > 2$ such that the matrix

$$\frac{\|M_{\alpha,\beta}\|}{\sqrt{\tau'}} < 1,$$

then $\frac{M_{\alpha,\beta}}{\tau'}$ is nonnegative, $I - \frac{\|M_{\alpha,\beta}\|}{\tau'}$ is non singular and

$$\left(I - \frac{M_{\alpha,\beta}}{\sqrt{\tau'}}\right)^{-1} = I + \frac{M_{\alpha,\beta}}{\sqrt{\tau'}} + \frac{M^2_{\alpha,\beta}}{\tau'} + \dots$$

From Lemma 3.1, we obtain that $\frac{M_{\alpha,\beta}}{\sqrt{\tau'}}$ converges to zero. As a consequence of Perov's fixed point theorem, N has a unique fixed $(x,y) \in S_b(\phi) \times S_b(\phi)$ which is the unique solution of problem (1.1). Let us denote this solution by (x,y).

5. An example

We consider the following impulsive neutral stochastic partial differential equation with Poisson jumps and finite delays driven by a Rosenblatt process of the form:

Example 5.1. *Consider the following couple stochastic partial differential equation with impulsive effects*

$$\begin{cases} d(x(t,\xi) & + & \frac{\alpha_1}{M_{\frac{1}{4}}\|(-A)^{\frac{3}{4}}\|}(x(t-u(t),\xi)+y(t-u(t),\xi)) = \frac{\partial^2}{\partial\xi^2}x(t,\xi) \\ & + & \alpha_2(x(t-r(t),\xi)+y(t-r(t),\xi))+e^{-t}dZ^H(t) \\ & + & \int_{\mathcal{Z}}\alpha_3\kappa(x(t-\rho(t),\xi)+y(t-\rho(t),\xi)\widetilde{N}(dt,d\kappa), \quad t\geq 0, \quad t\neq t_k, \quad 0\leq\xi\leq\pi, \\ d(y(t,\xi) & + & \frac{\lambda_1}{M_{\frac{1}{4}}\|(-A)^{\frac{3}{4}}\|}(x(t-u(t),\xi)-y(t-u(t),\xi)) = \frac{\partial^2}{\partial\xi^2}x(t,\xi) \\ & + & \lambda_2(x(t-r(t),\xi)-y(t-r(t),\xi))+e^{-t}dZ^H(t) \\ & + & \int_{\mathcal{Z}}\lambda_3\kappa(x(t-\rho(t),\xi)-y(t-\rho(t),\xi)\widetilde{N}(dt,d\kappa), \quad t\geq 0, \quad t\neq t_k, \quad 0\leq\xi\leq\pi, \\ x(t_k^+,\xi) & - & x(t_k^-,\xi)=\frac{\alpha_4}{2}x(t_k^-,\xi), \quad k=1,\cdots,m, \\ y(t_k^+,\xi) & - & y(t_k^-,\xi)=\frac{\bar{\lambda}_4}{2}y(t_k^-,\xi), \quad k=1,\cdots,m, \\ x(t,0) & = & x(t,\pi)=0, t\geq 0, \ \ \alpha_i,\lambda_i>0, \quad i=1,2,3,4 \\ y(t,0) & = & y(t,\pi)=0, t\geq 0, \\ x(s,\xi) & = & \phi_1(s,\xi), \ 0\leq\xi\leq\pi, \ \ -\tau\leq s\leq 0 \\ y(s,\xi) & = & \phi_2(s,\xi), \ 0\leq\xi\leq\pi, \end{cases} \tag{5.11}$$

Take $Y = X = L^2([0,\pi])$. We define the operator $A_1 = A_2 = A$ by $Au = u''$, with domain $D(A) = \{u \in X, u', u'' \in X \quad \text{and} \quad u(0) = u(\pi) = 0\}$.

Then, it is well known that

$$Ax = \sum_{n=1}^{\infty} n^2 \langle x, e_n\rangle_X e_n, \ x \in D(A),$$

and A is the infinitesimal generator of an analytic semigroup $\{S(t)\}_{t\geq 0}$ on X, which is given by

$S(t)u = \sum_{n=1}^{\infty} e^{-n^2t}\langle u, e_n\rangle e_n, \ u \in X,$ and $e_n(u) = (2/\pi)^{1/2}\sin(nu), n = 1, 2, \cdots,$ is the orthogonal set of eigenvectors of $-A$.

The bounded linear operator $(-A)^{\frac{3}{4}}$ *is given by*

$$(-A)^{\frac{3}{4}}x = \sum_{n=1}^{\infty} n^{\frac{3}{2}}\langle x, e_n\rangle_X e_n,$$

with domain

$$D((-A)^{\frac{3}{4}}) = \{x \in X,\ \sum_{n=1}^{\infty} n^{\frac{3}{2}}\langle x, e_n\rangle_X e_n \in X\}$$

The analytic semigroup $\{S(t)\}_{t>0}$, $t \in J$, *is compact, and there exists a constant* $M \geq 1$ *such that* $\|S(t)\|^2 \leq M$.

$Z^H(t)$ *is Rosenblatt process with parameter* $H \in (1/2, 1)$ *defined on a complete probability space* $(\Omega, \mathcal{F}, \mathbb{P})$. *In order to define the operator* $Q : \mathcal{K} \longrightarrow \mathcal{K}$, *we choose a sequence* $\{\sigma_n\}_{n\geq 1} \subset \mathbb{R}^+$, *set* $Qe_n = \sigma_n e_n$, *and assume that*

$$tr(Q) = \sum_{n=1}^{\infty} \sqrt{\sigma_n} < \infty.$$

Define the process $B_Q^H(s)$ *by*

$$Z^H(t) = \sum_{n=1}^{\infty} \sqrt{\sigma_n}\gamma_n^H(t)e_n,$$

where $H \in (1/2, 1)$, *and* $\{\gamma_n^H\}_{n\in\mathbb{N}}$ *is a sequence of two-sided one-dimensional mutually independent fractional Brownian motions.Now, rewrite (5.11) into the abstract form of (1.1). In order to model the problem (5.11) in the abstract form of (1.1), we consider the mapping* f^i, g^i *and* h^i *for each* $i = 1, 2$ *as follows*

$$g^1(t, x(t-u(t)), y(t-u(t))) = \frac{\alpha_1}{M_{\frac{1}{4}}\|(A)^{\frac{3}{4}}\|}(x(t-u(t)) + y(t-u(t))),$$

$$g^2(t, x(t-u(t)), y(t-u(t))) = \frac{\alpha_1}{M_{\frac{1}{4}}\|(A)^{\frac{3}{4}}\|}(x(t-u(t)) - y(t-u(t))),$$

and

$$f^1(t, x(t-u(t)), y(t-u(t))) = \alpha_2(x(t-r(t)) + y(t-r(t))),$$

$$f^2(t, x(t-u(t)), y(t-u(t))) = \lambda_2(x(t-r(t)) - y(t-r(t)))$$

and

$$h^1(t, x(t-\rho(t)), y(t-\rho(t)), \kappa) = \alpha_3\kappa(x(t-\rho(t)) + y(t-\rho(t)),$$

and

$$h^2(t, x(t-\rho(t)), y(t-\rho(t)), \kappa) = \lambda_3\kappa(x(t-\rho(t)) - y(t-\rho(t))$$

More precisely, f^i, g^i *and* h^i *satisfy Lipschitz condition with* $\|(-A)^{\frac{3}{4}}\| = 1$ $L_{f_1} = L_{\bar{f}_1} = \alpha_2, L_{f_2} = L_{\bar{f}_2} = \lambda_2$ *and* $L_{g_1} = L_{\bar{g}_1} = \alpha_2, L_{g_2} = L_{\bar{g}_2} = \frac{\alpha_1}{M_{\frac{1}{4}}}$, $L_{h_1} = L_{\bar{h}_1} = \int_{\mathcal{Z}} \alpha_3^2\kappa^2\nu(d\kappa)$ *and* $c_1 = \bar{c}_1 = \frac{\alpha_4}{2}$, $c_2 = \bar{c}_2 = \frac{\lambda_4}{2}$.*Thanks to these assumptions, it is straightforward to check* *that* $(H1)-(H6)$ *hold true and, then, assumptions in Theorem 4.1 are fulfilled, and we can conclude that system (5.11) possesses a mild solution on* $[-\tau, b]$.

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