International Congress on Differential Geometry and Related Topics

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BOOK OF ABSTRACTS

Draft

Ege University

İzmir, Türkiye

simadp.com/congress/dgrt • dgrtcongress@gmail.com

Foreword

Differential geometry, broadly constructed as the discipline that synthesizes the analytical apparatus of calculus and linear algebra with the synthetic tradition of geometry, has occupied a central position in the mathematical sciences for more than two centuries. Its conceptual evolution has been inextricably linked with some of the most profound transformations in our understanding of space, time, and the physical world.

The systematic foundations of the modern theory may be traced to the seminal contributions of Carl Friedrich Gauss, whose works established the differential geometry of curves and surfaces as a rigorous and autonomous mathematical discipline. Gauss introduced the notion of intrinsic curvature and demonstrated, through his celebrated Theorema Egregium, that the Gaussian curvature of a surface constitutes an intrinsic invariant—dependent solely upon the metric structure of the surface itself and invariant under isometric deformations of its embedding in ambient Euclidean space. The recognition that geometric properties could be investigated intrinsically, without reference to an external spatial framework, represented a paradigm shift of far-reaching consequence and may be regarded as the conceptual point of departure for all subsequent developments in the field.

Concurrent with these advances, a development of comparable significance was unfolding in the domain of axiomatic geometry. Lobachevsky, working in relative isolation at the University of Kazan, and Bolyai, a Hungarian officer whose investigations were conducted independently, succeeded in constructing consistent geometrical systems in which Euclid's parallel postulate is replaced by an alternative hypothesis. These independent discoveries of what would come to be known as non-Euclidean geometry constituted a decisive rupture with the two-millennial authority of the Euclidean framework and furnished the first compelling evidence that the geometry of physical space is not a matter of a priori necessity but rather an empirical question susceptible to mathematical and physical investigation.

The decisive generalization was achieved by Riemann, who, in his Habilitationsschrift of 1854, entitled Über die Hypothesen, welche der Geometrie zu Grunde liegen ("On the Hypotheses Which Lie at the Bases of Geometry"), introduced the concept of an n-dimensional manifold endowed with a Riemannian metric—a smoothly varying positive-definite bilinear form defined on the tangent space at each point. By extending Gauss's intrinsic geometry from surfaces to differentiable manifolds of arbitrary dimension, Riemann provided the comprehensive framework within which the geometry of space itself could be described intrinsically, in full generality, and without any embedding assumption.

It was precisely within the Riemannian framework that Albert Einstein, in collaboration with and building upon the mathematical groundwork of Grossmann, Levi-Civita, and Ricci-Curbastro, formulated the general theory of relativity in 1915. Einstein perceived that the phenomenon of gravitation admits of a geometric interpretation: the gravitational field is identified with the curvature of a four-dimensional Lorentzian manifold representing spacetime, whose metric tensor is determined, by means of the Einstein field equations, by the distribution of mass and energy. The geodesic equations of semi-Riemannian geometry thus became the equations of motion for test particles and light rays, while the curvature tensor assumed the role of the fundamental descriptor of gravitational phenomena. The unification of geometry and gravitation thereby achieved stands as one of the most consequential intellectual achievements of the twentieth century and inaugurated an enduring dialogue between differential geometry and theoretical physics that continues to the present day.

The interaction between differential geometry and gravitational physics has, in more recent decades, given rise to the study of lightlike (null) geometry, a subdiscipline of considerable importance in the analysis of black hole spacetimes and related phenomena. The event horizon of a black hole constitutes a null hypersurface—a submanifold whose normal vector is everywhere lightlike—and the causal structure of spacetime in the neighbourhood of singularities is governed by the geometry of null geodesics, null hypersurfaces, and degenerate semi-Riemannian structures. The rigidity theorems of black hole physics, the geometric analysis of Killing horizons, and the propagation of wavefronts in curved spacetime all rely in an essential manner on the techniques and results of lightlike differential geometry. The investigation of null submanifolds, degenerate metric structures, and their invariants has thus emerged as a vigorous and rapidly developing field of research, situated at the interface of pure differential geometry and mathematical relativity.

In a development that illustrates the remarkable range and adaptability of the manifold concept, the ideas of Riemannian geometry have, in recent years, found application in an unexpected domain: the theory of machine learning. The subfield known as manifold learning rests upon the hypothesis that data residing in a high-dimensional ambient space frequently lie on or near a low-dimensional manifold of significantly smaller intrinsic dimension. Algorithms such as ISOMAP, Locally Linear Embedding (LLE), Laplacian Eigenmaps, and t-Distributed Stochastic Neighbor Embedding (t-SNE) seek to recover this intrinsic manifold structure by exploiting local metric and topological properties of the data, thereby achieving dimensionality reduction that preserves the salient geometric features of the underlying distribution. That the same notions—intrinsic distance, geodesic structure, and local-to-global geometric reasoning—which Gauss introduced for the study of surfaces and Riemann generalized to abstract spaces, should now serve as the theoretical foundation for the analysis of high-dimensional data, constitutes a striking testament to the universality and enduring vitality of the manifold paradigm.

INTERNATIONAL CONGRESS ON DIFFERENTIAL GEOMETRY AND RELATED TOPICS (DGRT) is conceived in the spirit of this scholarly tradition. Its programme encompasses the classical geometry of curves and surfaces, the foundational theories of Riemannian and semi-Riemannian manifolds, the geometry of lightlike structures and their applications in mathematical physics, and the emerging intersections of differential geometry with data science and machine learning. It is our earnest hope that the intellectual exchange among participants—encompassing both established researchers and early-career scholars—will illuminate longstanding problems, generate novel directions of inquiry, and sustain the creative dialogue between geometry and the natural sciences that has animated this discipline since its inception.

We would like to express our sincere gratitude to Prof. Dr. Musa ALCI, Rector of Ege University, for his generous support and invaluable contribution to the successful organization of this congress.

We wish to extend our sincere gratitude to the invited speakers, Organizing Committee members, Scientific Committee members, Local Organizing Committee members and participants, and all those whose efforts and contributions have rendered this congress possible.

Prof. Dr. Bayram ŞAHİN
Chair of the Congress Organizing Committee

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Gabriel-Eduard Vîlcu

Gheorghe Mihoc-Caius Iocab Institute of Mathematical Statistics and Applied Mathematics of the Romanian Academy, Bucharest, Romania

National University of Science and Technology Politehnica Bucharest, Department of Mathematics and Informatics, Bucharest, Romania

On the Stability of Space Forms

In this talk, we discuss stability properties of geometric structures arising in almost contact metric geometry, with a particular emphasis on space forms. We begin by examining almost contact metric manifolds that are locally conformal to almost cosymplectic manifolds, where we derive stability results for the identity map under the assumption of pointwise constant ϕ-holomorphic sectional curvature. Motivated by these findings, we then study the stability of generalized S-space forms. We establish a general stability criterion and identify conditions that lead to instability. Several consequences of these results are discussed, including applications to $f.pk$-space forms, S-space forms, C-space forms, and Sasakian space forms. The presentation is primarily based on joint works with Cătălin Gherghe, J.-I. Inoguchi, and C.-D. Neacşu.

Keywords: Harmonic map, stability, generalized S-space form, Sasakian space form.

Mathematics Subject Classification (2020): 53C55, 53C43, 58E20.

Johan Gielis

Departmen of Biosciences Engineering, University of Antwerp, Belgium

Geniaal BV, Antwerp, Belgium

Universal Natural Shapes – New Models for the Sciences

In his Habilitationsschrift, Riemann mentioned the geometric tangent two-dimensional indicatrix $x^4 + y^4$ as a relevant extension of the Euclidean unit circle, thus conceiving the simplest Riemann–Finsler metrics. However, in 1818, Gabriel Lamé (1795–1860) had already provided a systematic treatment of supercircles and superellipses, with the aim of describing crystals geometrically. The generalization of Lamé curves to the Superformula, a generic geometric transformation generalizing conic sections, turns out to be an excellent scientific tool for the study of natural shapes and phenomena, from the very small to the very large. These developments have found their way in numerous applications in technology and science. It also connects many fields of science in a simple way, for example linking form and development, and inequality in economics and biology. In a dynamic and kinematic setting, recent developments include a complete differential geometry apparatus, adapted to these shapes. This aligns with Lamé's Unique Rational Science: find the best fitting coordinate systems and then solve the relevant boundary value problems.

Keywords: Superellipses, superformula, natural shapes.

Ejaz Ahmed

Brock University, Canada

Beyond Sparsity: Post-Shrinkage Estimation and Prediction in High-Dimensional Probability Models with Weak Signals

In many contemporary high-dimensional applications, the underlying signal structure is often more complex than the classical sparsity paradigm suggests. While only a small subset of predictors may exhibit strong effects, a large number of variables frequently contribute weak but collectively important signals. Standard penalized estimation and variable selection procedures are effective in identifying dominant predictors; however, they often overlook weak signals, leading to suboptimal estimation and prediction performance.
In this talk, we develop a post-shrinkage framework for high-dimensional regression models that explicitly accounts for the presence of weak signals. The proposed methodology combines the strengths of sparse model selection with post-selection adjustment, yielding improved prediction accuracy and enhanced estimation efficiency. We establish the theoretical properties of the proposed estimators through asymptotic analysis and characterize their behavior under both sparse and weakly sparse signal regimes.
Theoretical expressions for bias and risk are established and critically assessed under a range of high-dimensional settings. Extensive simulation studies demonstrate that the proposed post-shrinkage strategy consistently outperforms conventional penalized estimators and several state-of-the-art machine learning approaches across a broad range of high-dimensional settings. Applications to real high-dimensional datasets further illustrate the practical advantages of the method and highlight the importance of incorporating weak signals beyond traditional variable selection frameworks.
The talk concludes with a discussion of several open problems at the intersection of high-dimensional statistics, predictive modeling, and weak-signal learning, together with preliminary results that suggest promising directions for future research.

Miroslava Antić

University of Belgrade, Serbia

The Geometry of Certain Classes of Almost Complex Surfaces in $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$

This talk studies non-degenerate almost complex surfaces in the pseudo-nearly Kähler manifold $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$. We consider surfaces with constant isotropy and constant sectional curvature and examine the role of the almost product structure P in the geometry of such submanifolds. In particular, we analyze almost complex surfaces for which P maps the tangent bundle into the normal bundle. When the second fundamental form h is non-null, we obtain a classification of these surfaces and derive explicit local immersion formulae. Moreover, under the assumption of constant sectional curvature, we show that such surfaces must be flat and give their explicit immersion. In the case where the second fundamental form h is null, we analyze the resulting geometric conditions and describe the corresponding class of surfaces.

Keywords: Pseudo-nearly Kähler $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$, almost complex surface, almost product structure.

Mathematics Subject Classification (2020): 53A15, 53B25.

Contents

Abstracts of Oral Presentations

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Geometric Investigation of Real-Parameterized Rectifying Curves in Dual Spaces

Ayşenur Akbulut[1], Alper Osman Ogrenmis[2]

[1] *Graduate School of Natural and Applied Sciences, Fırat University, Elazığ, Türkiye,*
aysenurakbulut4444@gmail.com

[2] *Department of Mathematics, Faculty of Science, Fırat University, Elazığ, Türkiye,*
aogrenmis@firat.edu.tr

Abstract. Rectifying curves are an important class of space curves whose position vectors always lie in their rectifying planes. In this study, we investigate rectifying curves in the three-dimensional dual space by considering real-parameterized dual curves. Using the dual Frenet frame, we derive necessary and sufficient conditions for a dual curve to be rectifying. It is shown that a dual curve is rectifying if and only if its real component is a rectifying curve in the Euclidean space and its dual component satisfies a system of differential conditions involving the curvature and torsion of the real curve. Several examples are also presented.

Keywords: Dual curve, Rectifying curve, Dual space, Frenet frame.
Mathematics Subject Classification (2020): 53A04, 53A17.

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INTERNATIONAL CONGRESS ON Differential Geometry AND RELATED TOPICS

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Geodesics of Tumor Progression: An Information-Geometric Analysis of Pan-Cancer Transcriptomic Data

Ömer Akgüller[1,2], Mehmet Ali Balcı[2], Gizem Çalıbaşı Koçal[3]

[1] *Oncology Department, Institute of Health Sciences, Dokuz Eylül University, 35340 Izmir, Turkey,* omer.akguller@ogr.deu.edu.tr

[2] *Mathematics Department, Faculty of Science, Mugla Sitki Kocman University, 48000 Mugla, Turkey,* oakguller@mu.edu.tr, mehmetalibalci@mu.edu.tr

[3] *Translational Oncology Department, Institute of Oncology, Dokuz Eylül University, 35340 Izmir, Turkey,* gizem.calibasi@deu.edu.tr

Abstract. The transcriptomic profiles of biological samples do not uniformly occupy the high-dimensional gene-expression space but rather reside on a low-dimensional manifold governed by underlying regulatory networks. In this study, we present an information-geometric framework to analyze this transcriptomic manifold in the context of oncology. By equipping the parametric space of stationary gene expression distributions with the Fisher information metric, we rigorously quantify the statistical distinguishability of complex biological states. We model developmental and progressive trajectories as geodesics, which minimize the total information change. Furthermore, we evaluate the Riemannian curvature of the manifold to determine state stability: negative Ricci curvature corresponds to developmental bifurcations, while positive Ricci curvature indicates stable cellular attractors. We computationally demonstrate these theoretical concepts by mapping the transcriptomic landscapes of seven major cancer cohorts from The Cancer Genome Atlas. Our geometric approach successfully translates Waddington's epigenetic landscape into a rigorous mathematical structure, offering quantifiable insights into tumor heterogeneity and the stability of cancer states.

Keywords: Information Geometry, Transcriptomic Manifold, Fisher Information Metric, Riemannian Curvature, Systems Oncology.
Mathematics Subject Classification (2020): 53B20, 92C42, 94A17.

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Some notes on screen invariant lightlike hypersurfaces of locally product-like statistical manifolds

Ömer Aksu[1], Mehmet Gülbahar[2]

[1]*Department of Mathematics, Faculty of Science and Arts, Harran University, Şanlıurfa, Turkey,*
omraksu@hotmail.com

[2]*Department of Mathematics, Faculty of Science and Arts, Harran University, Şanlıurfa, Turkey,*
mehmetgulbahar@harran.edu.tr

Abstract. Some examples of screen invariant lightlike hypersurfaces of locally product-like statistical manifolds are presented. Certain specially defined vector fields capable of characterizing a semi-Riemannian manifold are investigated on these lightlike hypersurfaces. Various results are obtained using these types of vector fields.

Keywords: Lightlike hypersurface, Statistical manifold, Semi-Riemann manifold.
Mathematics Subject Classification (2020): 53C05, 53C25, 53C55.

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A Framed-Frame Based Formulation of Rolling Motion of Rigid Objects With Point Contact

Tuba Akyıldız[1], Bayram Şahin[2]

[1] *Department of Mathematics, Ege University, İzmir, Türkiye*, tubaakyildiz@gmail.com
[2] *Department of Mathematics, Ege University, İzmir, Türkiye*, bayram.sahin@gmail.com

Abstract. In this study, the pure rolling motion of two rigid objects over each other via a single point of contact is investigated from a geometric perspective. By relaxing the regularity assumption of the curve initially accepted in classical rolling theory, the contact curves are allowed to possess singular point or points in this work. Accordingly, the geometric modeling of the motion is reformulated using the theory of framed curves, a modern branch of differential geometry.

Keywords: Framed Curve, Singular points, Rigid Objects.
Mathematics Subject Classification (2020): 53A04, 53A05, 53A17, 58K05.

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On the Weak $\widetilde{\phi}$-invariance of the Ricci Tensor for g-natural Contact Metric Structures on Unit Tangent Sphere Bundles

Murat Altunbaş[1]

[1] *Department of Mathematics, Erzincan Binali Yıldırım University, Erzincan, Türkiye,*
maltunbas@erzincan.edu.tr

Abstract. We study the weak $\widetilde{\phi}$-invariance of the Ricci tensor on the unit tangent sphere bundle T_1M of a Riemannian manifold (M, g) equipped with a g-natural contact metric structure. We assume that M has dimension $m > 3$ and constant sectional curvature κ. In this case, on the contact distribution the Ricci tensor of $(T_1M, \widetilde{G})$ is determined by three scalars, and weak $\widetilde{\phi}$-invariance holds if and only if these scalars satisfy two linear relations. This gives two algebraic equations in κ whose coefficients depend on m and on the parameters a, b, c of the metric. For $b = 0$ the equations have exactly two solutions, and for the standard contact metric structure we recover a known result on unit tangent sphere bundles.

Keywords: Unit tangent sphere bundle, g-natural metric, contact metric structure, weakly $\widetilde{\phi}$-invariant Ricci tensor, space form.
Mathematics Subject Classification (2020): 53D15, 53C25, 53C15.

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A Classification of Curve Pairs Sharing Natural Darboux Planes on Oriented Surfaces

Gamze Aral[1], Pınar Balkı Okullu[2]

[1] *Manisa Celal Bayar University, Faculty of Engineering and Natural Sciences, Department of Mathematics, Manisa, Türkiye*, aralgamze055@gmail.com

[2] *Manisa Celal Bayar University, Faculty of Engineering and Natural Sciences, Department of Mathematics, Manisa, Türkiye*, pinar.balkiokullu@cbu.edu.tr

Abstract. In this paper, we investigate pairs of regular curves lying on the same oriented surface by means of the natural planes determined by the Darboux frame. First, three natural Darboux planes associated with the Darboux frame are introduced and their geometric properties are discussed. Then, all possible coincidence configurations between the natural Darboux planes of two curves are classified into nine distinct cases. For each case, the relations between the corresponding Darboux frame vectors are established, and the associated relations among the geodesic curvature, normal curvature and geodesic torsion are derived. Consequently, necessary conditions characterizing curve pairs sharing a common Darboux plane are obtained. The proposed approach provides a systematic framework for studying geometric relationships between curves on oriented surfaces through the natural planes of the Darboux frame.

Keywords: Darboux frame, natural Darboux planes, geodesic curvature, normal curvature, geodesic torsion, curves on surfaces.
Mathematics Subject Classification (2020):

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Adaptive Segmentation of Semi-Transparent Fluid Suspensions via Parametric Active Contours and Manifold Manipulation

Fatih Mustafa Atkeydiren[1], Arif Gürsoy[2]

[1]*Department of Computer Science, Institute of Sciences, Ege University, İzmir, Türkiye,* atkeydiren@gmail.com

[2]*Department of Computer Science, Ege University, İzmir, Türkiye,* arif.gursoy@ege.edu.tr

Abstract. Isolating translucent fluid boundaries is a key challenge in computer vision, as classical edge detectors may become unreliable in weak-gradient, optically transmissive media. We propose a hybrid geometric framework: the region of interest is first bounded via the Minimum Enclosing Circle problem; a localized Riemannian geometry-inspired transformation in the LAB color space then enhances weak gradients via CLAHE; finally, a parametric active contour (Snake) minimizes its energy functional to conform to the fluid boundary. A spatial CNN trained on the resulting segmented data reduced the mean 5-fold cross-validation loss from 0.2124 to 0.1522, suggesting the potential synergy between differential geometry and deep learning.

Keywords: Parametric Active Contours, Snakes, Riemannian Geometry in Machine Learning, Manifold Transformation, Convolutional Neural Networks.
Mathematics Subject Classification (2020): 53A04, 68U10.

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INTERNATIONAL CONGRESS ON
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Pseudo-Kähler Structures on Bicrossed Products

Gülhan Ayar[1]

[1] *Duzce University*, gulhanayar@duzce.edu.tr

Abstract. Pseudo-Kähler Lie algebras play an important role in differential geometry, Lie theory, and mathematical physics, particularly in the study of homogeneous pseudo-Riemannian manifolds, Ricci-flat metrics, and hypersymplectic geometry. Recent work by Conti and Gil-García has demonstrated that semidirect products provide an effective study for constructing new pseudo-Kähler Lie algebras from existing ones. However, semidirect products describe only one-sided actions between Lie algebras.

In this work, we propose a natural generalization of this construction by replacing semidirect products with bicrossed products arising from matched pairs of Lie algebras. Starting from two pseudo-Kähler Lie algebras equipped with mutually compatible derivation representations, we investigate the existence of invariant pseudo-Kähler structures on their bicrossed product.

Keywords: Pseudo-Kähler Lie algebras, Bicrossed products, Matched pairs of Lie algebras.

Mathematics Subject Classification (2020): 17B30, 53C15, 53C25.

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On Curves in Riemannian Manifolds Associated with Torse-Forming Vector Fields

Muhittin Evren Aydın[1], Esra Dilmen[2], Büşra Karakaya[3]

[1] *Department of Mathematics, Faculty of Science, Firat University*, meaydin@firat.edu.tr
[2] *Graduate School of Natural and Applied Sciences, Mathematics, Firat University*, esradlmn23@gmail.com

[3] *Graduate School of Natural and Applied Sciences, Mathematics, Firat University*, busrakarakaya404@gmail.com

Abstract.
Let ∇ be the Levi-Civita connection on a Riemannian manifold M. It is known from [7] that a vector field $\mathcal{V} \in \mathfrak{X}(M)$ is said to be torse-forming if the following holds

$$\nabla_X \mathcal{V} = fX + \omega(X)\mathcal{V},$$

where f and ω are the so-called potential function and generting form of $\mathcal{V}$, respectively. In this talk, we consider smooth curves Σ in Riemannian 3-manifolds when its principal unit normal $\mathcal{N}$ makes a prescribed angle with a torse-forming vector field $\mathcal{V}$ defined along the curve. Such curves are characterized in terms of the potential function f and the prescribed angle function. In particular cases, we give classification results.

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Keywords: Torse-forming vector field, hypersurface, ruled surface.
Mathematics Subject Classification (2020): 53B20, 53C42.

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Surfaces in Riemannian Manifolds by means of Torse-Forming Vector Fields

Muhittin Evren Aydın[1], Esra Dilmen[2], Büşra Karakaya[3]

[1] *Department of Mathematics, Faculty of Science, Firat University*, meaydin@firat.edu.tr
[2] *Graduate School of Natural and Applied Sciences, Mathematics, Firat University*, esradlmn23@gmail.com
[3] *Graduate School of Natural and Applied Sciences, Mathematics, Firat University*, busrakarakaya404@gmail.com

Abstract. Let M be a Riemannian manifold and ∇ the Levi-Civita connection on M. A vector field $\mathcal{V} \in \mathfrak{X}(M)$ is said to be torse-forming if we have the expression $\nabla_X \mathcal{V} = fX + \omega(X)\mathcal{V}$, where f and ω are the so-called potential function and generting form of $\mathcal{V}$, respectively. In this talk, we consider orientable surfaces Σ in a Riemannian 3-manifold M when its unit normal $\mathcal{N}$ makes a prescribed angle θ with a torse-forming vector field $\mathcal{V}$ defined along $\Sigma \subset M$. We characterize such surfaces in terms of the potential function f and the prescribed angle function θ. In some special situations, we also obtain classification results.

Acknowledgments: This study was supported by Scientific and Technological Research Council of Turkey (TUBITAK) under the Grant Number (123F451). The authors thank to TUBITAK for their supports.

Keywords: Torse-forming vector field, helix, curvature, torsion.
Mathematics Subject Classification (2020): 53B20, 53C42.

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Some Characterizations of $\zeta(Ric)$-vector fields and Their Applications to Ricci Solitons

Sibel Gerdan Aydın[1], Hakan Mete Taştan[2], Moctar Traore[3]

[1] *Department of Mathematics, İstanbul University, Vezneciler, 34134, İstanbul, Turkey,*
sibel.gerdan@istanbul.edu.tr
[2] *Department of Mathematics, İstanbul University, Vezneciler, 34134, İstanbul, Turkey,*
hakmete@istanbul.edu.tr
[3] *West institute of Mathematics of Gamal Abdel Nasser University of Conakry, Conakry, Guinea,*
moctar.traore@ogr.iu.edu.tr

Abstract. In this study, we demonstrate that every $\zeta(Ric)$-vector field on a Riemannian manifold is closed. Based on this, we provide new characterizations for Riemannian manifolds that admit these vector fields. We also examine the specific conditions of compactness and orientability. Furthermore, we explore the rigidity of Ricci solitons when their potential vector fields are of the $\zeta(Ric)$ type. A key result of our work is proving that the potential vector fields of steady gradient Ricci solitons are always $\zeta(Ric)$. To support our theoretical findings, we provide several illustrative examples. Finally, we discuss the presence of Ricci solitons on submanifolds equipped with $\zeta(Ric)$-vector fields.

Keywords: ζ(Ric)-vector field, compact and orientable Riemannian manifold, Ricci soliton, harmonic vector field.
Mathematics Subject Classification (2020): 53C20, 53C25, 54D30.

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The Equations of Geodesics on Helicoidal Surfaces in 4D Euclidean Space

Funda Babaarslan[1], Murat Babaarslan[2]

[1] *Faculty of Science and Letters, Department of Mathematics, 66900, Yozgat, Türkiye,*
`funda.kocabiyik@bozok.edu.tr`

[2] *Faculty of Science and Letters, Department of Mathematics, 66900, Yozgat, Türkiye,*
`murat.babaarslan@bozok.edu.tr`

Abstract. Geodesics are curves that represent locally shortest paths between two points on a surface and they can be characterized as critical points of the length or energy functionals in the calculus of variations. However, obtaining an analytical description of geodesics on different classes of surfaces is often a challenging mathematical problem. This problem lies at the intersection of differential geometry, the calculus of variations and functional analysis, and it has attracted considerable attention in the literature [1, 3]. The theoretical and computational study of geodesics has focused mainly on rotational and helicoidal surfaces because of their rich geometric structures (see, [1, 2, 4]). In recent years, increasing interest in higher–dimensional differential geometry has made the 4D Euclidean space, an important area of research. Unlike the rotations in 3D Euclidean space, which preserve an axis, the rotations in 4D Euclidean space preserve a plane, leading to a richer family of rotational and helicoidal surfaces.

In this talk, we investigate geodesics of helicoidal surfaces in 4D Euclidean space. Using the Euler–Lagrange equations, we derive the integral equations that characterize these geodesics. The obtained results extend existing studies from 3D Euclidean space to the 4D setting and contribute to a better understanding of the intrinsic geometry of these important classes of surfaces.

Keywords: Geodesic, Helicoidal surface, 4D Euclidean space.
Mathematics Subject Classification (2020): 53B25.

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Geometric Product and Rotation on R^n using Lie Product

Şenay Baydaş[1], Bülent Karakaş[2]

[1] *Van Yüzüncü Yıl University, Van, Türkiye*, sbaydas@yyu.edu.tr
[2] *Bartın University, Bartın, Türkiye*, bk@bartin.edu.tr

Abstract. In this study, the geometric product for rotations in R^n was interpreted using a new approach involving the Lie product on the set of A_n, $n \times n$ skew-symmetric matrices, $n = m(m-1)/2$, $m \geq 3$.

Keywords: Lie product, rotation, Clifford algebra, skew-symmetric matrices.
Mathematics Subject Classification (2020): 15A66, 15A67.

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Classification of Codazzi and Umbilical Hypersurfaces in $\mathrm{SL}_2(\mathbb{R}) \times \mathbb{R}$

Mohamed Belkhelfa[1], Hichem Mokni[2]

[1] *Faculty of Exact Sciences*
Mustapha Stambouli University
Mascara, 29000, Algeria, mohamed.belkhelfa@gmail.com
[2] *College of Sciences*
Taibah University
Al-Madinah Al-Munawarah, 41411, Saudi Arabia, hichemmokni@gmail.com

Abstract. We give the classification of totally geodesic, totally umbilical, parallel and Codazzi hypersurfaces in $\mathrm{SL}_2(\mathbb{R}) \times \mathbb{R}$

Keywords: Codazzi hypersurface, totally umbilical hypersurface, parallel hypersurfaces.
Mathematics Subject Classification (2020): MSC 53C30, 53C42.

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Some Results on Harmonic Maps into Principal Bundles and Generalized Magnetic Mappings

Habib Benziadi[1], Antonio López Almorox [2], Carlos Tejero Prieto[3]

[1,2,3] *Departamento de Matemáticas, Universidad de Salamanca, Plaza de la Merced 4, 37008, Salamanca, Spain*

[1,3] *Instituto de Física Fundamental y Matemáticas, Universidad de Salamanca, Plaza de la Merced 4, 37008, Salamanca, Spain,*

[1]habib.benziadi@usal.es, [2]alm@usal.es, [3]carlost@usal.es.

Abstract. In this talk, we develop a geometric framework for harmonic maps into principal G-bundles endowed with Kaluza-Klein metrics. Such maps are called Kaluza-Klein harmonic maps and naturally lead to the notion of generalized magnetic mappings for an arbitrary gauge group G. We show the influence of the geometry of the fibers of principal bundle on the Euler-Lagrange equations of Kaluza-Klein harmonic maps. As an application, we provide the natural generalization of the classical example of geodesic curves. Finally, we prove that, since the identity map of a Kaluza-Klein principal bundle is harmonic, its canonical projection is an uncharged generalized magnetic map.

Keywords: Kaluza-Klein harmonic maps, Generalized magnetic mappings.

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Classification of dual plane curves with constant curvature

Nursemin Çavdar[1], Muhittin Evren Aydın[2], Mahmut Ergüt[3]

[1] *Department of Mathematics, Tekirdağ Namık Kemal Üniversitesi*, nursemin_cavdar@hotmail.com
[2] *Department of Mathematics, Fırat Üniversitesi*, meaydin@firat.edu.tr
[3] *Department of Mathematics, Tekirdağ Namık Kemal Üniversitesi*, mergut@nku.edu.tr

Abstract. In this talk, we consider the problem of finding dual plane curves with constant curvature. This problem is solved in dual equiaffine plane and dual Lorentz-Minkowski plane. We also present examples supporting our examples.

Keywords: dual affine plane, dual Lorentz-Minkowski plane, dual curvature.
Mathematics Subject Classification (2020): 53A04, 53A15.

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Semi-Slant Submersions From Globally Conformal Kenmotsu Manifolds

Çağrıhan Çimen[1,3], Beran Pirinççi[2] Hakan Mete Taştan[2]

[1] *Department of Mathematics, Marmara University, İstanbul, Türkiye,*
cagrihan.cimen@marmara.edu.tr
[2] *Department of Mathematics, İstanbul University, İstanbul, Türkiye,*
beranp@istanbul.edu.tr, hakmete@istanbul.edu.tr
[3] *Institute of Graduate Studies in Sciences, İstanbul University, İstanbul, Türkiye*

Abstract. We study semi-slant submersions from globally conformal Kenmotsu manifolds. First, we derive the fundamental equations for such submersions. We then establish the conditions under which the distributions involved in the definition of the submersion are integrable. Finally, we investigate the conditions for the vertical and horizontal distributions to be totally geodesic.

Keywords: globally conformal Kenmotsu manifolds, Riemannian submersions, contact manifolds.
Mathematics Subject Classification (2020): 53C18, 53C20.

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Quasi Hemi-Slant Riemannian Maps from Sasakian Manifolds to Riemannian Manifolds

Ramazan Demir[1], Ahmet Özaydın[2], Sema Kazan[3], Cumali Yıldırım[4]

[1] *Department of Mathematics, Faculty of Arts and Sciences, Inonu University, 44000 Malatya, Türkiye*, ramazan.demir@inonu.edu.tr
[2] *Department of Mathematics, Faculty of Arts and Sciences, Inonu University, 44000 Malatya, Türkiye*, ahmtozydn241@icloud.com
[3] *Department of Mathematics, Faculty of Arts and Sciences, Inonu University, 44000 Malatya, Türkiye*, sema.bulut@inonu.edu.tr
[4] *Department of Mathematics, Faculty of Arts and Sciences, Inonu University, 44000 Malatya, Türkiye*, cumali.yildirim@inonu.edu.tr

Abstract. In this paper, we introduce a new class of Riemannian maps, called quasi hemi-slant Riemannian maps, from Sasakian manifolds to Riemannian manifolds. We investigate their geometric properties by studying the invariant, anti-invariant, and slant distributions naturally associated with these maps. We establish necessary and sufficient conditions for the integrability of these distributions and determine when they define totally geodesic foliations. Furthermore, we obtain characterizations of totally geodesic and harmonic quasi hemi-slant Riemannian maps. Finally, several nontrivial examples are presented to illustrate the theory and support the main results.

Keywords: Semi-slant Riemannian maps, Hemi-slant Riemannian maps, Quasi hemi-slant Riemannian maps, Almost contact metric manifolds, Sasakian manifolds.
Mathematics Subject Classification (2020): 53C15, 53B20, 53D15.

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Classification of Almost Ricci Solitons on Class-A Spacelike Hypersurfaces in Robertson-Walker Spacetimes

Burcu Bektaş Demirci[1], Nurettin Cenk Turgay[2], Sinem Güler [3]

[1] *Department of Software Engineering, Faculty of Engineering, Fatih Sultan Vakıf University, Istanbul 34010, Türkiye*, bbektas@fsm.edu.tr
[2] *Department of Mathematics, Faculty of Science and Letters, Istanbul Technical University, Istanbul 34469, Türkiye*, turgayn@itu.edu.tr
[3] *Department of Industrial Engineering, Istanbul Sabahattin Zaim University, Istanbul 34303, Türkiye*, sinem.guler@izu.edu.tr

Abstract. The tuple $(g, \xi, \eta, \beta, \gamma, \delta)$ defines a generalized soliton on a Riemannian manifold (M, g) if

$$\frac{1}{2}\mathcal{L}_\xi g + \beta \cdot \mathrm{Ric} = \gamma \cdot g + \delta \cdot \eta \otimes \eta,$$

where β, γ and δ are smooth functions on M. Here, it is assumed that $\eta = \xi^\flat$ is the dual 1-form of the potential vector field ξ, [1]. Depending on the choices of β, γ and δ, the generalized soliton reduces to a Ricci soliton or a Yamabe soliton.
In this work, we study almost Ricci solitons on spacelike hypersurfaces of Robertson-Walker spacetimes whose potential field is the projection of the comoving observer field on the tangent space of the hypersurfaces. Also, we give classifications of almost Ricci solitons on class A hypersurface of Robertson-Walker spacetimes.

Keywords: Robertson-Walker Spacetimes, Almost Ricci solitons, Class A hypersurfaces.
Mathematics Subject Classification (2020): 53C40, 53C42.
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A Note On Ricci-Yamabe Solitons

Sena Nur Doğdu[1], Erol Yaşar[2], Şemsi Eken Meriç [3]

[1] *Department of Mathematics, Mersin University, Mersin, Türkiye*, senanurdogdu@mersin.edu.tr
[2] *Department of Mathematics, Mersin University, Mersin, Türkiye*, yerol@mersin.edu.tr
[3] *Department of Mathematics, Mersin University, Mersin, Türkiye*, semsieken@mersin.edu.tr

Abstract. In this paper, we deal with a Riemannian map between Riemannian manifolds whose base manifold admits a Ricci-Yamabe soliton. Here, we calculate the Ricci tensors of $RangeF_*$ and $KerF_*$ and obtain some characterizations for which any fibre of $KerF_*$ and $RangeF_*$.

Keywords: Riemannian manifolds, Ricci-Yamabe solitons, Curvature.
Mathematics Subject Classification (2020): 53C25, 53C43.

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Geometric Properties of Hypersurfaces in Potent Riemannian Manifolds

İrem Erdeyer[1], Feyza Esra Erdoğan[2]

[1] *Ege University, Department of Mathematics, Izmir, Turkey*, erdeyer@gmail.com
[2] *Ege University, Department of Mathematics, Izmir, Turkey*, feyza.esra.erdogan@ege.edu.tr

Abstract. In this paper, we investigate the geometry of hypersurfaces in potent Riemannian manifolds endowed with a potent structure B satisfying $B^2 = B$. By decomposing the ambient potent structure along a real hypersurface, we construct the induced structure $\Sigma = (\tilde{B}, g, \hat{u}, \xi, a)$ and derive its fundamental algebraic and differential relations using the Gauss and Weingarten formulas. We study invariant, non-invariant, totally geodesic, minimal, and umbilical hypersurfaces. Assuming that the ambient potent structure is parallel with respect to the Levi-Civita connection, we prove that a non-invariant hypersurface is totally geodesic if and only if the induced tensor field, the induced vector field, and the associated 1-form are parallel. Furthermore, we characterize umbilical hypersurfaces and prove that if the induced tensor satisfies $\tilde{B} = aI$, then the hypersurface is necessarily either invariant or non-invariant totally geodesic. These results extend classical hypersurface characterizations known for metallic and almost product geometries to the framework of potent Riemannian manifolds.

Keywords: Potent manifolds, hypersurfaces of potent manifolds, invariant hypersurfaces.
Mathematics Subject Classification (2020): 53C40, 53C15.

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Mannheim Curves in 4-Dimensional Euclidean Space

Gülsüm Ezim[1], Bayram Şahin[2]
[1] *Ege University, Faculty of Science, Department of Mathematics, Izmir, Türkiye*
[2] *Ege University, Faculty of Science, Department of Mathematics, Izmir, Türkiye,*
`bayram.sahin@ege.edu.tr`

Abstract. The theory of curves in higher-dimensional Euclidean spaces has been a prominent subject of research in classical differential geometry, particularly regarding the generalization of special curve pairs. Among these, Mannheim curves and their partners are well-known for the unique geometric property where the principal normal vector of the first curve coincides with the binormal vector of the second curve at corresponding points. While the characterizations of these curves have been extensively detailed in 3-dimensional space, their existence and behavior in higher dimensions remain a critical question. In this study, we rigorously investigate the existence of Mannheim curves in 4-dimensional Euclidean space. By utilizing the Frenet frame apparatus and analyzing the fundamental geometric relationships between the Frenet curvatures, we construct a system of differential equations governed by the Mannheim definition. We examine the algebraic and geometric constraints imposed by these curvature functions in 4 dimensions and demonstrate that the strict geometric conditions required to maintain a Mannheim partner relation.

Keywords: Mannheim curves, 4-dimensional Euclidean space, Frenet frame, Mannheim partner curves, Curvature.
Mathematics Subject Classification (2020): 53A04, 53A05.

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Some basic properties of Yamabe soliton and Applications

Tanveer Fatima[1]

Department of Mathematics and Statistics, College of Science in Yanbu, Taibah University-Yanbu Governate, Saudi Arabia, tansari@taibahu.edu.sa

Abstract.
In this article, we show that to a Yamabe soliton $(N, h, \mathbf{u}, \lambda)$, there is a naturally associated skew-symmetric $(1, 1)$ tensor field ϕ called the associated tensor of the Yamabe soliton. It is shown that a connected Yamabe soliton $(N, h, \mathbf{u}, \lambda)$ whose scalar curvature τ is constant along the integral curves of the potential vector field $\mathbf{u}$ and associated tensor ϕ invariant under potential vector field $\mathbf{u}$ necessarily implies that h is a Yamabe metric. Also, knowing that the scalar curvature of the Euclidean space E^n is zero, in this section, we seek conditions under which a complete and connected Yamabe soliton $(N, h, \mathbf{u}, \lambda)$ having scalar curvature $\tau = 0$ is necessarily isometric to E^n. We achieve this result through demanding $\lambda \neq 0$ and the squared length of the covariant derivative of the potential vector field $\mathbf{u}$ has a proper lower bound. In a next result, we consider the case the scalar curvature $\tau \neq 0$ and obtain condition under which the metric h of the Yamabe soliton $(N, h, \mathbf{u}, \lambda)$ is a Yamabe metric. Also, we ask the potential vector field $\mathbf{u}$ of a Yamabe soliton $(N, h, \mathbf{u}, \lambda)$ to be a 2-Killing vector field and find conditions under which the metric h is a Yamabe metric. Finally, we consider gradient Yamabe soliton $(N, h, \nabla f, \lambda)$ and classify $(N, h, \nabla f, \lambda)$ whose potential vector field ∇f is eigenvector of the Ricci operator Q corresponding to constant eigenvalue μ.

Keywords: Yamabe soliton; Yamabe metric; associated tensor; 2-Killing vector field; gradient Yamabe soliton; Euclidean space

Mathematics Subject Classification (2020): 53E20, 53C21, 53C24.

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INTERNATIONAL CONGRESS ON Differential Geometry AND RELATED TOPICS

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On Some Parallel Surfaces Generated by Revolution Surfaces in Galilean 3-Space

İsmet Gölgeleyen

Department of Mathematics, Faculty of Science, Zonguldak Bülent Ecevit University, Zonguldak 67100, Türkiye, ismet.golgeleyen@beun.edu.tr

Abstract. Parallel surfaces constitute an important topic in differential geometry, as they provide insight into the behavior of geometric invariants under parallel displacement. In this study, we investigate the parallel surfaces associated with the three classes of revolution surfaces in Galilean 3-space. Our aim is to characterize those revolution surfaces whose parallel counterparts possess constant mean curvature. To this end, explicit formulas for the mean curvature of the parallel surfaces are established, and the corresponding ordinary differential equations governing the generating profile curves are obtained. Solving these equations yields explicit parametrizations of the resulting surface families. Several representative examples are also presented and visualized to illustrate the geometric features of the constructed parallel surfaces and to demonstrate the effect of the parallel surface transformation in the Galilean framework.

Keywords: Galilean 3-space, Parallel surface, Revolution surface, Mean curvature.
Mathematics Subject Classification (2020): 53A10, 53A35.

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INTERNATIONAL CONGRESS ON Differential Geometry AND RELATED TOPICS

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Constant-Chord Curves Generated by Complementary Antipodal Radial Functions

Deniz Güçler[1]

[1] *Canadian Institute of Technology, Tirana, Albania*, deniz.gucler@yahoo.com

Abstract. In this work, we consider a family of closed planar curves defined in polar coordinates through radial functions satisfying a complementary symmetry condition between antipodal angular positions. We show that this symmetry naturally induces a constant-chord property between opposite points of the curve and gives rise to a broad class of non-circular closed profiles. Representative examples are presented to demonstrate the diversity of shapes that can be obtained within this family. The results provide a geometric framework for the study of constant-chord curves and suggest possible connections with topics in differential geometry, geometric design, and related areas of applied mathematics.

Keywords: Constant-Chord Curves, Closed Planar Curves, Antipodal Symmetry.
Mathematics Subject Classification (2020): 53A04, 51M04.

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Generalized Quasi-Einstein Weyl Manifolds with a Ricci Quarter-Symmetric Connection

İlhan Gül[1]

[1] *Faculty of Engineering and Natural Sciences, Maltepe University, İstanbul, Türkiye*
ilhangul@maltepe.edu.tr

Abstract. We study generalized quasi-Einstein Weyl manifolds endowed with a Ricci quarter-symmetric connection. Our aim is to examine the geometric properties arising from the interaction between the Weyl structure and this connection. We discuss the Ricci tensor, certain special vector fields, and the relations between the associated 1-forms. We also investigate the behavior of the structure under generalized concircular and related conformal mappings. These considerations contribute to the study of Einstein-type structures and non-metric connections in Weyl geometry.

Keywords: Generalized quasi-Einstein Weyl manifold, Ricci quarter-symmetric connection.
Mathematics Subject Classification (2020): 53C18, 53C25, 53B05.

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Interpolating Sesqui-Harmonic Minimal Curves in S-space Forms

Ece Güler[1], Şerife Nur Bozdağ[2]

[1] *Ege University*, ece.guler2005@gmail.com

[2] *Ege University*, serife.nur.yalcin@ege.edu.tr

Abstract. In this study, the concept of interpolating sesqui-harmonic minimal curves is investigated in S-space forms. First, necessary and sufficient conditions are established for a Legendre curve to be an interpolating sesqui-harmonic minimal curve in S-space forms. These conditions are summarized in a main theorem. The obtained conditions are then investigated under several special cases determined by the relationships between the parameters c and s, and by whether certain vector fields of the S-space form are orthogonal or parallel. The results obtained for each of these special cases are presented in the form of theorems.

Keywords: S-Space Form, Legendre Curves, Biharmonic Curves, Harmonic Curves.
Mathematics Subject Classification (2020): 53C25, 53C43.

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Non-Existence of Gradient Ricci-Yamabe Solitons on Siklos Spacetimes

Sinem Güler[1]

[1] *Department of Industrial Engineering, Istanbul Sabahattin Zaim University, Istanbul, Türkiye,*
sinem.guler@izu.edu.tr

Abstract. This study addresses Siklos spacetimes, an important class of spacetime models providing a solution to the Einstein field equations with a negative cosmological constant [1, 2]. First, we introduce certain typical soliton equations that arise in the theory of Ricci and Yamabe flows [3]. Subsequently, we examine the geometric properties of Siklos spacetimes and prove that a gradient Ricci-Yamabe soliton cannot exist in such a spacetime. This non-existence theorem raises a significant question: *Which solitons do exist in Siklos spacetimes?* Consequently, we aim to answer this question.

Keywords: Siklos metric, Gradient Yamabe soliton, Gradient Ricci-Yamabe soliton.
Mathematics Subject Classification (2020): 53C50, 53B302.
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Recent Studies on Bézier and PH Curves

Mehmet Gümüş

Lapseki Vocational School, Çanakkale Onsekiz Mart University, Çanakkale, Türkiye,
mehmetgumus@comu.edu.tr

Abstract. Curve theory is important for both mathematics, physics and applied sciences. Rapid advancements in differential geometry since the 17th century have also influenced technological developments. Bézier and PH-curves play a significant role, particularly in computer aided geometric design. The Bézier curve is named after French engineer Pierre Bézier (1910–1999), who used it in the 1960s for designing curves for the bodywork of Renault cars. Farouki and Sakkalis defined Pythagorean-hodograph (PH) curves in 1994. Many studies have been conducted on these two curves since the 2000s. Some of these include: Gür Soğat and Gümüş, "On Bertrand and Spherical PH-Curves in Minkowski 3-Space" [1]; Özbal and Gümüş, "PH-Helical Curves in Minkowski Space" [2]; Doğan and Gümüş, "On Quadratic Bézier PH-Curves in Euclidean and Minkowski Spaces" [3]; Mollaoğulları et al., "A New Method to Obtain PH-Helical Curves in E^{n+1}" [4]; Yang Shen and Xingxuan Peng, "Geometric Characteristics and Constructions of Cubic Indirect-PH Curves" [5]; Fenhong Li, Gang Hu, Muhammad Abbas and Kenjiro T. Miura, "The Generalized H-Bézier Model: Geometric Continuity Conditions and Applications to Curve and Surface Modeling" [6]. The aim of this study is to summarize the recent studies on Bézier and PH-curves and to compile the latest research.

Keywords: Differential geometry, Bézier curve, PH-curve.
Mathematics Subject Classification (2020): 53A04, 53Z50.

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INTERNATIONAL CONGRESS ON Differential Geometry AND RELATED TOPICS

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Timelike Curves that are Slant Submanifolds

Yılmaz Gündüzalp

Department of Mathematics, Dicle University, 21280, Sur, Diyarbakır, Türkiye,
ygunduzalp@dicle.edu.tr

Abstract. We introduce the notion of slant submanifolds of a semi-Riemannian product manifold. We study their some properties. Then, we show that in a semi-Riemannian product manifold there exist timelike curves that are slant submanifolds. Also, we characterize these timelike curves, and study them in two and three-dimensional semi-Riemannian product manifolds.

Keywords: Slant submanifolds, timelike curve, semi-Riemannian product manifold.
Mathematics Subject Classification (2020): 53C15, 53C40.

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On the Geometric Structures of Non-Newtonian Calculi

Aykut Has[1], Beyhan Yılmaz[2]

[1] *Kahramanmaraş Sütçü Imam University, Faculty of Science, Department of Mathematics, Kahramanmaraş, Turkey*, ahas@ksu.edu.tr

[2] *Kahramanmaraş Sütçü Imam University, Faculty of Science, Department of Mathematics, Kahramanmaraş, Turkey*, beyhanyilmaz@ksu.edu.tr

Abstract. Non-Newtonian calculi are alternative approaches based on redefining the concepts of classical calculus through different algebraic structures and generator functions. These calculi provide new research opportunities not only in analysis but also in various disciplines such as differential equations, optimization, and geometry. In this presentation, after introducing the fundamental framework of Non-Newtonian calculi, the geometric structures developed within this framework will be discussed. The redefinition of fundamental geometric concepts in the context of Non-Newtonian calculi will be examined, and their similarities and differences with classical differential geometry will be evaluated.

Keywords: Non-Newtonian calculi, Multiplicative calculus.
Mathematics Subject Classification (2020): 53A04, 53A35, 26A24, 26E60.

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A New Approach to Mannheim Curve Using Multiplicative Calculus

Hatice Kübra İzci[1], Aykut Has[2], Beyhan Yılmaz[3], Kazım İlarslan[4]

[1,2,3]*Kahramanmaraş Sütçü Imam University, Faculty of Science, Department of Mathematics, Kahramanmaraş, Turkey*, kubraizci2@outlook.com, ahas@ksu.edu.tr, beyhanyilmaz@ksu.edu.tr

[4]*Kırıkkale University, Faculty of Sciences and Arts, Department of Mathematics, Kırıkkale, Turkey*, kilarslan@yahoo.com

Abstract. Special curves have been extensively studied in classical differential geometry. Among these, Mannheim curve pairs have attracted considerable attention, and a new approach to their characterization in the three-dimensional Euclidean space was proposed in [1]. Motivated by this study, we reconsider Mannheim curve pairs within the framework of multiplicative calculus, whose algebraic structure differs fundamentally from that of classical calculus. By employing the tools of multiplicative differential geometry, we establish a new formulation of Mannheim curves and derive several theoretical results and characterization theorems. The proposed approach extends the concept of Mannheim curve pairs to the multiplicative setting and provides a foundation for further studies on special curves in non-Newtonian geometric frameworks.

Keywords: Multiplicative calculus, Approach to Mannheim curve.
Mathematics Subject Classification (2020): 53A04, 53A35, 26E60.

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A Comparison of Human and Machine Thinking Through Sudoku-Solving Algorithms

Mert Kaptan[1], Fulya Şahin[2]

[1] *Department of Mathematics, Ege University, İzmir, Türkiye*, 04210000416@ogrenci.ege.edu.tr
[2] *Department of Mathematics, Ege University, İzmir, Türkiye*, fulya.sahin@ege.edu.tr

Abstract. This study compares the algorithmic solving process used by computer programs with the heuristic strategies employed by human solvers. In addition, solved Sudoku grids were examined from a linear algebraic perspective by computing their determinant values to investigate invertibility and to explore whether determinant values reflect structural properties of valid Sudoku solution matrices. A randomized recursive backtracking algorithm, combined with Fisher-Yates shuffling method, was implemented to generate and solve Sudoku grids, while the problem was also modeled as a Constraint Satisfaction Problem to reflect its algorithmic structure. In parallel, human solving strategies across varying difficulty levels ranging from basic techniques such as Naked and Hidden Singles to advanced graph-based patterns such as X-Wing and Swordfish, were examined to highlight the structural differences between machine search and human inference. Determinant values were computed for hundred solved 9×9 Sudoku matrices at the 9×9, 6×6 and 3×3 levels. Preliminary results show that all computed determinants of 9×9 matrices are nonzero, with values varying widely across matrices and sub-blocks. The examined sudoku matrices were all found to be invertible. The study is ongoing, further analysis is planned to generalize these findings across a larger dataset.

Keywords: Sudoku, Recursive Backtracking, Constraint Satisfaction Problem, Determinant.
Mathematics Subject Classification (2020): 05B15, 15A15, 68T20, 68W40.

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Hemi-slant submersions whose total manifolds are globally conformal Kenmotsu manifolds

Deniz Ulusoy Kaynar[1,3], Beran Pirinççi[2], Hakan Mete Taştan[2]

[1] *Department of Management Information Systems, İstinye University, İstanbul, Türkiye*, deniz.ulusoy@istinye.edu.tr
[2] *Department of Mathematics, İstanbul University, İstanbul, Türkiye*, beranp@istanbul.edu.tr, hakmete@istanbul.edu.tr
[3] *Institute of Graduate Studies in Science, İstanbul University, İstanbul, Türkiye*, deniz.ulusoy@ogr.iu.edu.tr

Abstract. In this talk, we investigate hemi-slant submersions whose total manifolds are globally conformal Kenmotsu manifolds. The fundamental equations characterizing these submersions are obtained. Moreover, the integrability of the distributions associated with the hemi-slant structure is examined, and necessary and sufficient conditions are presented. The study also characterizes when the vertical and horizontal distributions determine totally geodesic foliations.

Keywords: globally conformal Kenmotsu manifolds, Riemannian submersions, contact manifolds, hemi-slant Submersion.
Mathematics Subject Classification (2020): 53C18, 53C20.

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On the Construction of Control Nets for Translation Bézier Surfaces with Applications to Paraboloids and Parabolic in E^3

Şeyda Kılıçoğlu[1], Semra Yurttançıkmaz[2]

[1] *Başkent University, Faculty of Education*, seyda@baskent.edu.tr
[2] *Atatürk University, Faculty of Science*, semrakaya@atauni.edu.tr

Abstract. Building upon our previous work on lower-order Bézier surfaces, this paper extends the matrix representation framework to Bézier surfaces of arbitrary $(n \times m)$-th order generated by algebraic operators. The proposed approach provides a direct construction of surface control nets from the control points of the generating Bézier curves. By exploiting the algebraic properties of translation (additive sum) and cross product surfaces, explicit formulas for the corresponding control matrices are derived in terms of the vector operations applied to the control points of the constituent curves. To demonstrate the effectiveness of the proposed framework, several quadric surface examples in E^3 are presented. In particular, elliptic and hyperbolic paraboloids are represented as biquadratic and higher-order Bézier surfaces using both translation and cross product constructions, illustrating the geometric versatility and computational efficiency of the proposed matrix formulation for surface modeling.

Keywords: Bézier curve control points, Bézier surface control net, paraboloids control net.
Mathematics Subject Classification (2020): 53A04, 53A05.

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On the Construction of Control Nets for Cross Product Bézier Surfaces with Applications in E^3

Şeyda Kılıçoğlu[1]

[1] *Baskent University, Faculty of Education*, seyda@baskent.edu.tr

Abstract. Following our previous investigations of lower-order Bézier surfaces, this paper generalizes the matrix representation framework to Bézier surfaces of arbitrary $(n \times m)$-th order generated by algebraic operators. The main objective is to derive the control nets of these higher-order surfaces directly from the control points of the generating Bézier curves. Exploiting the algebraic structure of cross product surfaces, we establish that the control matrix of the resulting Bézier surface can be obtained explicitly through the cross product of the control points of the constituent curves. Finally, several examples in E^3 are presented to illustrate the geometric characteristics and computational effectiveness of the proposed approach.

Keywords: Bézier curve control points, Bézier surface control net, control net.

Mathematics Subject Classification (2020): 53A04, 53A05.

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INTERNATIONAL CONGRESS ON Differential Geometry AND RELATED TOPICS

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Information Geometric Metrics of Spherical and Hyperbolic Spaces in Two Dimensions

Ahmet Koltuksuz[1], Mert Hazar Hasgül[2]

[1] *Yasar University, Dept. of Computer Engineering*, ahmet.koltuksuz@yasar.edu.tr
[2] *Yasar University, Dept. of Computer Engineering*, mert.hasgul@yasar.edu.tr

Abstract. Traditionally, information has been recorded and processed in two-dimensional Euclidean spaces, except for biological structures. A recent study clearly demonstrated that the information can also be recorded and processed on an n-dimensional digital manifold [1][2]. Moreover, the n-dimensional digital manifold satisfies Shannon information metrics and entropy axioms [1]. On the other hand, from the perspective of information geometry, the two-dimensional spherical and hyperbolic geometries, as well as the manifolds, seem to satisfy the information metrics and entropic axioms [3][4]. Based on these considerations, the purpose of this study is to present the information geometric metrics of the two-dimensional spherical and hyperbolic geometries. The number of Euclidean, spherical, and hyperbolic surfaces, each with a different radius, were filled with geodesic triangles in computer simulations using the Python language and Numpy, Scipy and Matplotlib libraries to obtain the information metrics. In terms of entropy, while the spherical space is different, the closeness of Euclidean space to hyperbolic space is striking.

Keywords: Information geometry, spherical and hyperbolic geometry, entropy.
Mathematics Subject Classification (2020): 53B12, 94A17, 62B11.

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Almost Pseudo $\mathcal{B}$-symmetric Space-times in $F(r)$-gravity

Dilek Kurt[1], Hülya Bağdatlı Yılmaz [2]

[1] *Institute of Pure and Applied Sciences, Marmara University, Istanbul, Türkiye*
dilekyardim@marun.edu.tr

[2] *Department of Mathematics, Marmara University, Istanbul, Türkiye*
hbagdatli@marmara.edu.tr

Abstract.
In this work, almost pseudo $\mathcal{B}$-symmetric space-times are introduced and their geometric as well as physical consequences are investigated within the framework of $F(r)$-gravity.
Under the assumption of constant scalar curvature, the geometric structure induced by the $\mathcal{B}$ tensor is applied to modified gravitational models. In this context, several representative $F(r)$ models are examined in detail.
The corresponding modified field equations are derived, and the effective energy density and effective pressure are obtained. Furthermore, the standard energy conditions are analyzed, providing a clear interpretation of the physical viability of these models.
These results demonstrate that an almost pseudo $\mathcal{B}$-symmetric geometry provides a natural and effective framework for the analysis of modified gravitational models.

Keywords: Perfect fluid space-time, $\mathcal{B}$ tensor, Modified gravity, Energy conditions, Almost pseudo $\mathcal{B}$ symmetric manifolds.
Mathematics Subject Classification (2020): 53C25, 53C50, 83D05.

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Relationship between Jacobi-orthogonal and pseudo-Riemannian Osserman algebraic curvature tensors

Katarina Lukić

Faculty of Mathematics, University of Belgrade, Belgrade, Serbia, katarina.lukic@matf.bg.ac.rs

Abstract. Jacobi-orthogonal algebraic curvature tensors were introduced in [1] as a new potential characterization of Riemannian Osserman tensors. While it was proven there that any Jacobi-orthogonal tensor is Osserman, all known Osserman (two-root or Clifford) tensors are also Jacobi-orthogonal. We generalize the concept of Jacobi-orthogonality to indefinite scalar product spaces and investigate its relationship with Osserman algebraic curvature tensors. It was shown in [2] that any 3-dimensional algebraic curvature tensor is Jacobi-orthogonal if and only if it is of constant sectional curvature. Furthermore, it was established that a 4-dimensional Jacobi-diagonalizable algebraic curvature tensor is Jacobi-orthogonal if and only if it is Osserman.

Keywords: indefinite scalar product space, Osserman algebraic curvature tensor, Jacobi orthogonality.
Mathematics Subject Classification (2020): 53B30, 53B20.

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Some Geometric Properties of Para-Kenmotsu Manifolds

Tuğba Mert[1], Mehmet Atçeken[2], Emel Karaca[3]

[1] *Department of Mathematics, Sivas Cumhuriyet University, Sivas, Türkiye,*
tmert@cumhuriyet.edu.tr
[2] *Department of Mathematics, Aksaray University, Aksaray, Türkiye,*
mehmetatceken@aksaray.edu.tr
[3] *Department of Mathematics, Ankara Hacı Bayram Veli University, Ankara, Türkiye,*
emel.karaca@hbv.edu.tr

Abstract. The relationships between the Riemann, Ricci, W_0, and W_0^* curvature tensors on para-Kenmotsu manifolds are investigated in this work. Our goal is to determine the relationships between these tensors and the conditions under which specific geometric properties appear. The obtained results improve the current literature for para-Kenmotsu manifolds and offer novel perspectives for further research in both pure and applied differential geometry.

Keywords: Para-Kenmotsu manifold, Curvature tensor, Einstein manifold.
Mathematics Subject Classification (2020): 58A03, 58A05.

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Geodesic Regression on Riemannian Manifolds and Its Applications

Metehan Mertcan[1], Bayram Şahin[2]

[1] *Department of Mathematics, Faculty of Science, Ege University, İzmir, Türkiye,*
91240000884@ogrenci.ege.edu.tr

[2] *Department of Mathematics, Faculty of Science, Ege University, İzmir, Türkiye,*
bayram.sahin@ege.edu.tr

Abstract. Classical linear regression models the relationship between an independent variable and a dependent variable using a straight line. However, when the dependent variable takes values on a nonlinear manifold, the standard straight-line model in Euclidean space loses its validity. Geodesic regression provides a direct generalization of linear regression to this setting, where the relationship is modeled by a geodesic curve on the manifold. The model relies on two parameters, an initial point p and an initial velocity vector v. The estimation is formulated as a least-squares problem that minimizes the sum of squared geodesic distances from the data points to the model.
After introducing the fundamental rationale of the method and the least-squares framework, this study presents the manifold generalization of the R^2 statistic for measuring goodness of fit, alongside a permutation test to assess the significance of the model. The theoretical background of the research specifically the existence and uniqueness of the solution in symmetric spaces and the maximum likelihood interpretation is outlined in broad terms without explicit proofs. Finally, the method is implemented in detail on the sphere S^2, where the data points, the initial point, the velocity vector, and the geodesic prediction model are computed and visualized. Subsequently, the geodesic prediction model is also constructed and tested with data on the Poincaré upper half-plane.

Keywords: Geodesic regression, Riemannian manifolds, Least-squares estimation.
Mathematics Subject Classification (2020): 53C22, 58E10.

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Curvature invariants on partially totally real submanifolds in complex space forms

Ion Mihai

University of Bucharest, Romania, imihai@mi.unibuc.ro

Abstract. Kähler manifolds are the most studied complex manifolds. A Kähler manifold with constant holomorphic sectional curvature is said to be a complex space form. There are special classes of submanifolds in Kähler manifolds. Recently, Poyraz, Şahin and Yerlikaya [2] defined partially totally real submanifolds in Kähler manifolds. In the present paper, we study curvature invariants adapted to partially totally real submanifolds. First, we define a Riemannian invariant analogous to the CR δ-invariant introduced by Chen for anti-holomorphic submanifolds in complex space forms (see [1]) and estimate it in terms of the squared mean curvature. This invariant characterizes the geometric interaction between the totally real distribution and its orthogonal complementary distribution. Furthermore, we obtain Chen-Ricci inequalities that distinguish between tangent directions in the two distributions. Finally, we prove a Chen first inequality for totally real 2-plane sections.

Keywords: Kähler manifold; complex space form; partially totally real submanifold; δ-invariants, curvature inequalities.
Mathematics Subject Classification (2020): 53C25, 53C40.

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Θ-Statistical Manifolds and Θ-CR Submanifolds

Mirjana Milijevic[1], Luis Yapu[2]

[1] *University of Banja Luka, Faculty of Economics*, mirjana.milijevic@ef.unibl.org
[2] *Universidade Federal Fluminense*, luis.yapu@gmail.com

Abstract. We introduce and examine Θ-statistical manifolds, namely statistical manifolds (M, g, ∇, ∇^*) endowed with a non-zero $(1,1)$-tensor Θ satisfying the gauge equation $\nabla_X(\Theta Y) = \Theta\nabla^*_X Y$. The study characterizes this condition in terms of the statistical difference tensor K in the Levi-Civita parallel case, where $K_X(\Theta Y) = -\Theta K_X Y$, and records several algebraic subclasses, including complex, product, projection, skew-adjoint and symmetric types. We also introduce Θ-CR submanifolds by decomposing ΘY into tangent and normal components and by using the induced statistical and dual connections through the Gauss and Weingarten formulas. The framework extends holomorphic statistical geometry and offers a CR-type submanifold theory governed by a general gauge-compatible tensor.

Keywords: Statistical manifolds, gauge-compatible tensors, Θ-statistical manifolds.
Mathematics Subject Classification (2020): 53C15, 53C40.

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Cubic Projective Vector Fields on $\mathbb{RP}^3$

Abdigappar Narmanov[1], Shohruh Rahmonov[1]

[1] *Department of Mathematics, National University of Uzbekistan, Tashkent, Uzbekistan*
narmanov@yandex.com, rahmonovshohruh91@gmail.com

Abstract. We study a two-parameter family of smooth vector fields on the real projective space $\mathbb{RP}^3$ induced by antipodally odd cubic polynomial vector fields on the sphere $S^3 \subset \mathbb{R}^4$. The fields are defined by coupled rotations in the orthogonal coordinate planes (x_1, x_2) and (x_3, x_4), where each angular velocity depends on the squared radius in the complementary plane. We prove tangency to S^3 and compatibility with the antipodal involution; hence the family descends to $\mathbb{RP}^3$. We also compare it with the classical vector fields induced by linear actions on $\mathbb{R}^4$ and show that, except for $\alpha = \beta = 0$, it is not of linear-projective type. An explicit first integral $I = x_1^2 + x_2^2$ is obtained. Its regular level sets are invariant tori $T_c \cong S^1 \times S^1$, $0 < c < 1$, and on each torus the dynamics reduces to the linear flow $\dot{\theta}_1 = 1 + \alpha(1 - c)$, $\dot{\theta}_2 = 1 + \beta c$. This gives a complete classification of periodic and quasiperiodic trajectories according to the rationality of the frequency ratio. Finally, we determine the singular set and the bifurcation curve $\alpha\beta + \alpha + \beta = 0$, on which an invariant torus becomes a torus of equilibria. The construction provides an explicit nonlinear projective model with completely tractable toric dynamics.

Keywords: projective space, cubic vector field, invariant torus, first integral, bifurcation curve.
Mathematics Subject Classification (2020): 37C10, 37C27, 53A20, 58K45.

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Ricci Soliton structure on the cotangent bundle

Filiz Ocak

Department of Mathematics, Karadeniz Technical University, Trabzon, TÜRKİYE
filiz.ocak@ktu.edu.tr

Abstract. Let (M, h) be a Riemannian manifold, Y a vector field on M, and λ a real constant. The triple (M, Y, λ) is called a Ricci soliton if

$$Ric + \frac{1}{2} L_Y h = \lambda h,$$

where Ric denotes the Ricci tensor of h and $L_Y h$ is the Lie derivative of h along Y. A Ricci soliton is said to be steady, shrinking, or expanding according as $\lambda = 0$, $\lambda < 0$ or $\lambda > 0$, respectively. In this paper, we investigate Ricci solitons associated with certain lifts of vector fields on the cotangent bundle.

Keywords: Cotangent bundle, Ricci soliton.
Mathematics Subject Classification (2020): 53C25, 53A45.
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α-Metallic Metric Riemann Manifolds

Sümeyra Özakgül[1], Feyza Esra Erdoğan[2]

[1] *Mathematics Department, Ege University, Izmir, Türkiye*, ozsume45@hotmail.com
[2] *Mathematics Department, Ege University, Izmir, Türkiye*, feyza.esra.erdogan@ege.edu.tr

Abstract. In this paper, we investigate the curvature properties of α-metallic metric Riemann manifolds having constant sectional curvature. We first prove that, under certain conditions, an α-metallic metric Riemann manifold with constant (real) sectional curvature is necessarily flat. Motivated by this observation, we examine holomorphic-like and holomorphic bisectional-like sectional curvatures associated with these manifolds and show that both curvatures vanish identically. Since these curvature notions fail to provide a meaningful characterization of the geometry of α-metallic metric Riemann manifolds, we introduce a new curvature invariant, called the α-metallic metric sectional curvature, and present an illustrative example. Futhermore, we derive the expression of curvature tensor for α-metallic metric Riemann manifold with constant α-metallic metric sectional curvature. Finally, we establish a necessary and sufficient condition for such manifolds to be an Einstein.

Keywords: α-metallic metric Riemann manifold, α-metallic metric sectional curvature, curvature tensor, Einstein manifold curvature, holomorphic sectional curvature.
Mathematics Subject Classification (2020): 53C15, 53B20, 53C25.

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Towards a Differential Geometry of Superelliptic Manifolds: Curvature, Singularities and Geometric Structures

Zehra Özdemir[1], Johan Gielis[2]

[1]*Department of Mathematics, Faculty of Sciences, Ankara University, 06100 Ankara, Türkiye,*
zehrao@ankara.edu.tr

[2]*Department of Bioscience Engineering, University of Antwerp, Belgium,*
johan.gielis@gmail.com

Abstract. The superformula provides a unified framework to describe a wide variety of natural and geometric forms [1],[2]. Although its applications have been extensively explored in diverse disciplines, the study of the underlying differential geometric structures has only recently begun [3]. Motivated by the interplay between geometry, analysis, and applications, this talk aims to develop a differential geometric framework for superelliptic manifolds induced by superelliptic and superformula-based structures.

We introduce superelliptic manifolds and discuss generalized notions of metric structures, curvature, immersions, and embeddings arising from superelliptic unit geometries. In this setting, we investigate how classical concepts of differential geometry extend beyond the Euclidean unit circle and examine the role of singularities and their possible absorption within generalized geometric frameworks.

Inspired by the work of Bang-Yen Chen [4],[5] several geometric structures are considered in the superelliptic setting, including rectifying curves and submanifolds, finite-type structures, and biharmonic phenomena. Particular attention is given to the extent to which classical results remain valid under superelliptic geometries.

The talk concludes with perspectives toward generalized harmonic analysis on supershapes, superelliptic Fourier expansions, and future directions involving geometric flows and canonical vector fields. These developments suggest new connections between superformula-based geometry, differential geometry, and mathematical physics.

Keywords: Superelliptic manifolds; superformula; differential geometry; curvature; rectifying submanifolds; finite-type structures; biharmonic geometry; generalized Fourier analysis.

Mathematics Subject Classification (2020): 53C40, 53A04, 53C42, 58B20, 53C44.

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August 17–18, 2026
Ege University, İzmir, Türkiye

On the Existence of Strictly Locally Conformal Kenmotsu Structures

Beran Pirinççi[1], Hakan Mete Taştan[2]

[1] *Department of Mathematics, İstanbul University, Vezneciler, İstanbul, Turkey,*
beranp@istanbul.edu.tr

[2] *Department of Mathematics, İstanbul University, Vezneciler, İstanbul, Turkey,*
hakmete@istanbul.edu.tr

Abstract. In this presentation, we undertake a rigorous investigation into the existence of strictly locally conformally Kenmotsu manifolds. The aim of this study is to explore the geometric properties and conditions under which these strictly (non-globally) conformal structures arise. In order to provide substantiation for the theoretical findings, an illustrative example is constructed.

Keywords: almost contact metric manifolds, locally conformal almost contact metric structures, locally conformal Kenmotsu manifolds, Lee form.
Mathematics Subject Classification (2020): 53C18, 53C25.

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On variational inequalities on geometric structures

Ariana Pitea[1],

[1] *National University of Science and Technology Politehnica Bucharest*, ariana.pitea@upb.ro

Abstract. The talk emphasizes the role played by the geometric structure of Hadamard manifolds in the context of variational inequality problems. The aim is to show a numerical algorithm for the determination of a solution of such a class in a set of the common fixed points of generalized nonexpansive operators. A qualitative study of the numerical process is conducted and applications of the results are presented.

Keywords: variational inequality, Hadamard manifolds, normal S-iteration.
Mathematics Subject Classification (2020): 47J05, 47H09, 49J30, 58C30.

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Optimal Inequalities for Submanifolds in Locally Product Riemannian Manifolds with Constant Product Sectional Curvature

Gülistan Polat[1], Özlem Deniz[2], Bayram Şahin[3]

[1] *Department of Mathematics, Ege University, İzmir, Türkiye*, glstnyldrmm@gmail.com
[2] *Department of Mathematics, Ege University, İzmir, Türkiye*, ozlemdeniz612@gmail.com
[3] *Department of Mathematics, Ege University, İzmir, Türkiye*, bayram.sahin@ege.edu.tr

Abstract. In this study, locally product Riemannian manifolds with constant sectional curvature are investigated. It is shown that, under certain conditions, locally product Riemannian manifolds with constant real sectional curvature are flat. A notion of constant product sectional curvature is then introduced, together with the corresponding Riemann curvature tensor. The Einstein condition for this curvature structure is also examined. Finally, using this curvature tensor, optimal Chen and Casorati inequalities for submanifolds are established, and their equality cases are characterized geometrically.

Keywords: Locally product manifolds, Constant section curvature, Einstein manifolds, Chen inequality, Casorati curvatures.
Mathematics Subject Classification (2020): 53C40, 53C42, 53C25, 53B25.

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The Role of the Reeb Vector Field in Clairaut Semi-Invariant Riemannian Maps from Trans-Sasakian Manifolds

Murat Polat

Department of Mathematics Faculty of Science Dicle University Diyarbakır, Turkey.
murat.polat@dicle.edu.tr

Abstract. In this paper, we investigate Clairaut semi-invariant Riemannian maps from trans-Sasakian manifolds with particular emphasis on the geometric role of the Reeb vector field. We prove that the Clairaut structure exhibits two fundamentally different behaviors according to whether the Reeb vector field is vertical or horizontal. In the vertical case, the classical Clairaut geometry is recovered, whereas the horizontal case reveals additional contributions arising from the trans-Sasakian structure. These differences lead to distinct characterizations of the Clairaut condition together with corresponding consequences for harmonicity and curvature. The results provide a unified geometric interpretation of the interaction between Clairaut geometry and trans-Sasakian structures and extend the current theory of semi-invariant Riemannian maps.

Keywords: Clairaut condition, Riemannian map, Semi-invariant Riemannian map, Trans-Sasakian manifold, Reeb vector field.
Mathematics Subject Classification (2020): 53C15, 53C43, 53C25, 58E20.

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On Transversely Kähler Almost Contact Metric Hom-Lie Algebras

Deniz Poyraz[1,2], Bayram Şahin[2]

[1] *Department of Mathematics, Graduate School of Natural and Applied Sciences, Ege University,* deniz.poyraz@ege.edu.tr

[2] *Department of Mathematics, Faculty of Science, Ege University,* bayram.sahin@ege.edu.tr

Abstract. Hom-Lie algebras, introduced as a natural generalization of Lie algebras, provide a suitable framework for extending classical geometric structures. Motivated by the recent study of transversely Kähler almost contact metric Lie algebras, this work aims to introduce the notion of transversely Kähler almost contact metric Hom-Lie algebras. Specifically, almost contact metric Hom-Lie algebras with an involutive twisting map ($\alpha^2 = Id$) are considered. The fundamental definitions and properties of transversely Kähler almost contact metric structures are adapted to the Hom-Lie setting, and the associated Hom-type identities are formulated and investigated. This study is expected to provide a basis for further investigations of transversely Kähler structures on Hom-Lie algebras and contribute to the development of Hom-contact geometry.

Keywords: Keywords: Hom-Lie algebra, almost contact metric Hom-Lie algebra, transversely Kähler structure.
Mathematics Subject Classification (2020): 17A30, 53C15, 53C25, 53D15.

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Geometry of Clairaut Riemannian Warped Product Submersions

Arkadeepta Roy[1], Kiran Meena[2], Hemangi Madhusudan Shah[3]

[1,3] *Harish-Chandra Research Institute*, [1] arkadeeptaroy@hri.res.in
[1,3] *Homi Bhabha National Institute*, [3] hemangimshah@hri.res.in
[2] *Indian Institute of Technology Jodhpur*, kirankapishmeena@gmail.com

Abstract. In this talk, we discuss about Riemannian Warped Product Submersions, generalizing the concept of Riemannian submersions (introduced by Barret O'Neill in his paper "The Fundamental Equations of a Submersion") by incorporating a warping function. Recalling the definitions of Riemannian submersions, Clairaut submersions and warped product manifolds and some of their important properties, we will discuss the structure and properties of Riemannian warped product submersions, with a focus on the influence of the warping function on geometry of the base and fibers. We will explore conditions under which such submersions preserve key geometric properties, including totally geodesic fibers, totally umbilical fibers etc. Finally, we will talk about generalizing the idea of Clairaut submersions in the context of warped product manifolds, leading to potential directions for future research in this area. We explore detailed formulas for the Riemannian, Ricci, and sectional curvature tensors of the source space of such submersions in terms of the geometry of the target and fiber manifolds, and the warping and girth functions. These computations provide geometric insight into how warping and the Clairaut condition affect curvature properties, such as conformal flatness and the Einstein property. This talk is based on our paper *Geometry of Clairaut Riemannian Warped Product Submersion* [1].

Keywords: Clairaut submersions, Riemannian warped product submersions, Trivial warping, Harmonic maps, Conformal flatness, Einstein manifolds.
Mathematics Subject Classification (2020): 53B20, 53B25, 53C25, 53C35.

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Non-null curves in the semi-Euclidean space under isometric and conformal maps

Derya Sağlam[1], Umut Selvi[2]

[1] *Department of Mathematics, Ankara Hacı Bayram Veli University, Ankara, 06900, Türkiye,*
derya.saglam@hbv.edu.tr

[2] *Department of Mathematics, Ankara Hacı Bayram Veli University, Ankara, 06900, Türkiye,*
umut.selvi@hbv.edu.tr

Abstract. We study non-null rectifying, osculating and normal curves lying on non-degenerate smooth surfaces in the semi-Euclidean space $\mathbb{E}^4_\nu$, and we determine when each curve type is preserved by isometric, conformal and homothetic conformal maps between two such surfaces. The differential of such a map does not commute with second and higher order derivatives. This produces a correction vector along the image curve, and a curve type is preserved exactly when a suitable component of this correction vanishes, together with a residual term in the rectifying and osculating cases. All conditions are stated both at the frame level and in local coordinates, and they reduce to the known Euclidean results when the index is zero. We also prove that the second binormal osculating class is rigid, and we construct explicit examples in Minkowski and neutral signatures realizing four different causal subcases.

Keywords: semi-Euclidean space, non-null curve, conformal transformation.
Mathematics Subject Classification (2020): 53A04, 53A35, 53B30.

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Bounding Box Regression on Riemannian Manifolds for Object Tracking

Pelin Sancaktar[1], Necla Kircali Gürsoy[2] Arif Gürsoy[3]

[1] *Department of Mathematics, Graduate School of Natural and Applied Sciences, Ege University, Bornova, Izmir 35040, Turkiye*, 91250000696@ogrenci.ege.edu.tr

[2] *Department of Computer Programming, Ege University, Bornova, Izmir 35040, Turkiye*, necla.kircali.gursoy@ege.edu.tr

[3] *Department of Mathematics, Faculty of Science, Ege University, Bornova, Izmir 35040, Turkiye*, arif.gursoy@ege.edu.tr

Abstract. Euclidean metrics (IoU, CIoU) cause gradient vanishing and jitter for disjoint UAV targets. Modeling bounding boxes as 2D Gaussians and minimizing the 2-Wasserstein distance between their symmetric positive-definite (SPD) covariance matrices on a Riemannian manifold provides smooth gradient flows. This reduces YOLO's ID switches and oscillation. Despite Riemannian computational FPS drops, real-time tracking is maintained.

Keywords: Riemannian Manifold, Object Tracking, Bounding Box Regression, Wasserstein Distance, Information Geometry.
Mathematics Subject Classification (2020): 53B20, 53B12, 68T07, 68T45.

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On Hemi-slant Submersions from Locally Conformal Kähler Manifolds

Sezin Aykurt Sepet[1], Meltem Karaismailoğlu[2]

[1] *Department of Mathematics, Faculty of Science and Letters, Kırşehir Ahi Evran University, Kırşehir, Turkey*, saykurt@ahievran.edu.tr

[2] *Department of Mathematics, Faculty of Science and Letters, Kırşehir Ahi Evran University, Kırşehir, Turkey*, mltm.33.27@hotmail.com

Abstract. In this paper, we investigate hemi-slant submersions from global conformal Kähler manifolds. We first present an illustrative example and derive the fundamental equations associated with these submersions. We then establish several results concerning the integrability of the anti-invariant and slant distributions, together with conditions ensuring the total geodesicity of the corresponding foliations. Finally, we derive some optimal inequalities for hemi-slant submersions whose total manifolds are Hopf space forms.

Keywords: Locally conformal Kähler manifold, Hopf space form, Hemi-slant submersion.
Mathematics Subject Classification (2020): 53C15, 53B35.

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Ege University, İzmir, Türkiye

Some Lightlike Submanifolds of Meta-Golden Semi-Riemannian Manifolds

Müge Sert[1], Feyza Esra Erdoğan[2]

[1] *Department of Mathematics, Ege University, İzmir, Turkey*, mugesert9790ms@gmail.com
[2] *Department of Mathematics, Ege University, İzmir, Turkey*, feyza.esra.erdogan@ege.edu.tr

Abstract. In this study, we investigate invariant and screen transversal lightlike submanifolds of local meta-golden semi-Riemannian manifolds. Motivated by the meta-golden structure, which generalizes golden and metallic structures through the Meta-Golden Chi ratio, we examine the geometric properties of these two classes of lightlike submanifolds. Using the Duggal–Bejancu formalism, we obtain necessary and sufficient conditions concerning the integrability of the screen and radical distributions. We also derive characterizations of totally geodesic foliations and conditions for the induced connection to be metric. The results extend several known findings on golden and metallic semi-Riemannian manifolds and clarify the interaction between meta-golden structures and lightlike geometry.

Keywords: Meta-golden semi-Riemannian manifolds; invariant lightlike submanifolds; screen transversal lightlike submanifolds.
Mathematics Subject Classification (2020): 53C15, 57R15.

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August 17–18, 2026
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Optimal inequalities involving Casorati curvatures along Riemannian maps and Riemannian submersions for Quaternionic space forms

Ravindra Singh[1]

[1] *Department of Mathematics, Banaras Hindu University, Varanasi, India*, khandelrs@bhu.ac.in

Abstract. In this paper, we establish Casorati inequalities for Riemannian maps and Riemannian submersions on Quaternionic space forms, and we present geometric characterizations of the equality cases. First, we derive Casorati inequalities for Riemannian maps to Quaternionic space forms and describe the corresponding equality cases in terms of the geometric properties of the maps. Next, we obtain Casorati inequalities involving the fundamental tensor fields T and A for Riemannian submersions from Quaternionic space forms to Riemannian manifolds, together with their geometric interpretations. In particular, we show that the equality case of the inequalities associated with the tensor field A along the horizontal distribution is characterized, while the equality cases associated with the tensor field T along the vertical distribution, as well as with both T and A along the horizontal and vertical distributions, are also characterized.

Keywords: Riemannian submersions, Riemannian maps, Casorati invariants, Quaternionic space forms.
Mathematics Subject Classification (2020): 53B20, 53B35, 53C15, 53D15.

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On the Curvature Tensors of Metallic Semi-Riemannian Structures Induced by Almost Paracontact Metric Structures

Mehmet Solgun[1], Yasemin Karababa[1]

[1] *Bilecik Şeyh Edebali University, Faculty of Science, Department of Mathematics, Bilecik, Türkiye*
mehmet.solgun@bilecik.edu.tr, yaseminn2604@gmail.com

Abstract. In this study, the curvature properties of metallic semi-Riemannian structures induced by almost paracontact metric structures are investigated. Let $(M, \varphi, \xi, \eta, g)$ be a $(2n+1)$-dimensional almost paracontact metric manifold, and let $(M, \Psi, \widetilde{g})$ denote the metallic semi-Riemannian structure induced by (φ, ξ, η, g). By using the relations between the Levi-Civita connections associated with g and $\widetilde{g}$, the corresponding Riemann curvature tensors are examined. In particular, when the initial structure is para-Kenmotsu or para-Sasakian, explicit relations between the Ricci tensors of g and $\widetilde{g}$ are obtained by contracting the corresponding curvature identities. It is shown that, in the para-Kenmotsu case, the difference between the Ricci tensors is determined by the tensor $g - \eta \otimes \eta$, whereas in the para-Sasakian case, additional terms depending on the coefficients of the metallic structure arise. By taking the trace with respect to the induced metric $\widetilde{g}$, the corresponding scalar curvature relations are also obtained in both cases.

Keywords: Almost paracontact metric manifold; metallic semi-Riemannian structure; para-Kenmotsu manifold; para-Sasakian manifold; Ricci tensor; scalar curvature.

Mathematics Subject Classification (2020): 53C15, 53C25.

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Framed Surface Pencils with a Given Framed Geodesic

Sevilay Çoruh Şenocak[1], Ergin Bayram[2]

[1] *Ondokuz Mayıs University, Faculty of Science, Department of Mathematics, Samsun, Türkiye,* sevilay.coruhsenocak@omu.edu.tr

[2] *Ondokuz Mayıs University, Faculty of Science, Department of Mathematics, Samsun, Türkiye,* ergin.bayram@omu.edu.tr

Abstract. In this study, a surface pencil passing through a given framed curve is constructed in parametric form, and the conditions to obtain a framed surface are determined. The criteria for the framed curve to be a geodesic on the framed surface are also defined. This geodesic property is shown to be invariant under rotations and reflections. To illustrate the findings, two examples of a framed surface containing the given framed geodesic are presented.

Keywords: Framed curve, Framed surface, Surface pencil, Geodesic.
Mathematics Subject Classification (2020): 53A04, 53A05, 58K05.

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INTERNATIONAL CONGRESS ON
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Fibonacci Sequence in Bézier Curves

Sude Şenyaz[1], Bayram Şahin[2]

[1] *Ege University, Faculty of Science, Department of Mathematics, Izmir, Türkiye,*
senyaz.sude@gmail.com

[2] *Ege University, Faculty of Science, Department of Mathematics, Izmir, Türkiye,*
bayram.sahin@ege.edu.tr

Abstract. Bézier curves are among the most widely used parametric curves in computer-aided design, computer graphics, animation, and geometric modeling. They are defined by a set of control points together with Bernstein basis polynomials, enabling the construction of smooth, flexible, and easily controllable geometric shapes. Owing to their continuity, numerical stability, and intuitive shape control, Bézier curves have become one of the fundamental tools in curve design and geometric modeling. In recent years, considerable attention has also been devoted to extending classical Bézier curves by incorporating special numerical sequences and alternative coefficient structures.

In this study, the classical Bézier curve formulation is investigated by incorporating the Fibonacci sequence into its construction. The control points of the Bézier curve are generated using successive Fibonacci numbers, and the geometric behavior of the resulting curves is examined. Bézier curves of different degrees are constructed and compared to analyze the influence of the Fibonacci sequence on the shape, symmetry, curvature, and overall geometric characteristics of the curves. The findings demonstrate how the use of Fibonacci numbers provides an alternative perspective for constructing and analyzing Bézier curves and contributes to the development of novel approaches in parametric curve modeling.

Keywords: Fibonacci sequence, Bézier curve, Bernstein polynomials, Casteljau Algorithm.

Mathematics Subject Classification (2020): 53A04, 65D17.

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INTERNATIONAL CONGRESS ON Differential Geometry AND RELATED TOPICS

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Kenmotsu-like statistical manifolds and their submersions

Kazuhiko Takano[1], Jun-ichi Inoguchi[2]

[1] *Institute of Humanities, Shinshu University, Matsumoto, 390-8621, Japan,*
ktakano@shinshu-u.ac.jp

[2] *Department of Mathematics, Hokkaido University, Sapporo, 060-0810, Japan,*
inoguchi@math.sci.hokudai.ac.jp

Abstract. Let (M, g) be a semi-Riemannian manifold with an almost contact structure (φ, ξ, η). If M admits another tensor field φ^* of type $(1, 1)$ satisfying

$$g(\varphi E, F) + g(E, \varphi^* F) = 0$$

for any vector fields E and F on M, then (M, g) is called an almost contact metric manifold with the almost contact structure (φ, ξ, η). The pair (M, g) is an almost contact metric manifold with (φ, ξ, η) if and only if it is also an almost contact metric manifold with (φ^*, ξ, η). We introduce a Kenmotsu-like statistical manifold, which is an analogue of a Kenmotsu manifold. Then, (M, g, ∇) is a Kenmotsu-like statistical manifold with the almost contact structure (φ, ξ, η) if and only if (M, g, ∇^*) is equipped with the dual almost contact structure (φ^*, ξ, η). In this talk, we intruduce Kenmotsu-like statistical manifolds and their submersions ([1]). The first author has previously studied Sasaki-like statistical manifolds and their submersions in [2].

Keywords: Almost contact manifold, Statistical manifold, Kenmotsu-like statistical manifold, Statistical submersion, Kenmotsu-like statistical submersion.
Mathematics Subject Classification (2020): 53B05, 53B12, 53C15.

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A New Conformal Change on Almost Contact Metric Structures

Hakan Mete Taştan[1], Beran Pirinççi[2]

[1]*Department of Mathematics, İstanbul University, Vezneciler, İstanbul, Turkey,*
hakmete@istanbul.edu.tr
[2]*Department of Mathematics, İstanbul University, Vezneciler, İstanbul, Turkey,*
beranp@istanbul.edu.tr

Abstract. In this talk, we introduce a new conformal change that preserves almost contact metric structures. By means of this change, we define locally transversally conformal Kenmotsu manifolds, for which we derive fundamental equations and characterizations. Moreover, we present an explicit example to support our theoretical results.

Keywords: almost contact metric manifold, locally transversally conformal Kenmotsu manifold, transversal Lee form.
Mathematics Subject Classification (2020): 53C18, 53C25.

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Slant Distributions on Submanifolds of Para-Hermitian Manifolds

Mehmet Fatih Tezcanlar[1], Mehmet Gülbahar[2]

[1] *Department of Mathematics, Faculty of Science and Arts, Harran University, Şanlıurfa, Turkey,*
m.fatih_hoca@hotmail.com

[2] *Department of Mathematics, Faculty of Science and Arts, Harran University, Şanlıurfa, Turkey,*
mehmetgulbahar@harran.edu.tr

Abstract. In this study, some characterizations related to slant distributions of bi-slant submanifolds of para-Hermitian manifolds are given. Some conditions regarding the case when a bi-slant submanifold is a slant submanifold are presented.

Keywords: Bi-slant submanifold, Distribution, Para-Hermitian manifold.
Mathematics Subject Classification (2020): 53C42, 53C55.

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The Rectifying Slant Helices in Pseudo-Galilean Space

Aykut Toplama[1], Mustafa Dede[2], Cumali Ekici[3]

[1] *Kilis 7 Aralik University, Faculty of Science, Department of Mathematics,* aykut.toplama@kilis.edu.tr
[2] *Kilis 7 Aralik University, Faculty of Science, Department of Mathematics,* mustafadede@kilis.edu.tr
[3] *Eskisehir Osmangazi University, Faculty of Science, Department of Mathematics and Computer Science,* cekici@ogu.edu.tr

Abstract. In this paper, a new characterization of rectifying slant curves, a special class of curves in Pseudo-Galilean space, is constructed. Additionally, the characterization of this curve class is stated for a special case. Furthermore, this special curve obtained from an example is presented with a figure.

Keywords: Pseudo-Galilean Space, Curves, Non-Euclidean Geometry, Rectifying Curve, Slant Curve.
Mathematics Subject Classification (2020): 53A35, 53A04.

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Marginally Trapped Surfaces of a Warped Space-time with Lorentzian Fiber

Nurettin Cenk Turgay[1],

[1]*Istanbul Technical University*, turgayn@itu.edu.tr

Abstract. Let $L^3_1(c) = (\Omega_c, g_c)$ denote the three-dimensional Lorentzian space form of constant sectional curvature c, where

$$g_c = \frac{1}{\left(1 + \frac{c}{4}(-t^2 + x^2 + y^2)\right)^2} \left(-dt^2 + dx^2 + dy^2\right),$$

defined on

$$\Omega_c = \left\{(t, x, y) \in \mathbb{R}^3 : 1 + \frac{c}{4}(-t^2 + x^2 + y^2) > 0\right\}.$$

For an open interval $I \subset \mathbb{R}$ and a smooth function $f : I \to (0, \infty)$, we consider the warped space-time

$$\widetilde{M}^4_1 = L^3_1(c)_f \times I = (\Omega_c \times I, \widetilde{g}),$$

endowed with the metric

$$\widetilde{g} = f(z)^2 g_c + dz^2$$

and we let $\mathcal{T}$ denote the stress-energy tensor $\widetilde{M}^4_1$.
Note that a space-like surface immersed in $\widetilde{M}$ is called marginally trapped if its mean curvature vector field is lightlike at every point. In this talk, we establish several new results on the geometry of marginally trapped surfaces in $\widetilde{M}$ by considering tangential and normal components of ∂_z together with the tensor $\mathcal{T}$.

Keywords: marginally trapped surface, Keyword 2, Keyword 3.
Mathematics Subject Classification (2020): 53C42 (Primary), 53A10.

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Transformer Attention as a Statistical Manifold: Fisher–Rao Geometry and Head Similarity Analysis

İnan Ünal[1]

[1] *Department of Finance, Banking and Insurance, Vocational School of Tunceli, Munzur University, Tunceli, Türkiye,*

inanunal@munzur.edu.tr

Abstract. Self-attention matrices are row-stochastic structures whose rows define categorical probability distributions. This study models transformer attention as a statistical manifold and investigates its geometry using the Fisher–Rao metric. The attention space is represented as a product of probability simplices, and its tangent structure, metric, geodesic distance, and rank-based stratification are examined. To demonstrate the practical relevance of the framework, attention-head similarities in a pretrained BERT model are analyzed using Fisher–Rao and Frobenius distances. The results reveal non-homogeneous geometric organization among attention heads and identify distinct similarity patterns. These findings indicate that information geometry provides a principled framework for analyzing the probabilistic structure of transformer attention.

Keywords: Information geometry, Transformer, self-attention, Fisher–Rao metric, deep learning.
Mathematics Subject Classification (2020): 53B12, 62B10, 68T07.

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A Survey On Conformal Riemannian Maps

Şener Yanan
Adıyaman University, syanan@adiyaman.edu.tr

Abstract. Conformal Riemannian maps from almost Hermitian manifolds to Riemannian manifolds, namely conformal anti-invariant Riemannian maps, conformal semi-invariant Riemannian maps, conformal slant Riemannian maps, conformal hemi-slant Riemannian maps and conformal bi-slant Riemannian maps, are introduced. We mainly focus on their decompositions and proper examples of conformal Riemannian maps.

Keywords: Submersions, Conformal Riemannian Maps, Hermitian Manifolds.
Mathematics Subject Classification (2020): 53C18, 53C20, 53C55.

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Generalized Multiplicative Lorentzian Magnetic Flux Tubes

Ebru Yanık[1], Hazal Ceyhan[2], Zehra Özdemir[3]

[1,3] *Department of Mathematics, Science Faculty, Ankara University, 06100 Ankara, Türkiye*, ebruyanik@ankara.edu.tr, zehrao@ankara.edu.tr

[2] *Department of Fundamental Sciences, Faculty of Engineering and Architecture, İzmir Bakırçay University, 35665 İzmir, Türkiye*, hazal.ceyhan@bakircay.edu.tr

Abstract. Magnetic flux tubes, formed by the family of magnetic field lines threading a bounded cross-section, constitute a fundamental geometric object linking differential geometry to electromagnetism and plasma dynamics. In this work, we develop a framework for such tubes within Lorentzian geometry using multiplicative (non-Newtonian) calculus, in which differentiation and integration rest on multiplicative rather than additive operations. This generalization is natural for processes governed by exponential scaling and yields a Frenet–Serret apparatus whose multiplicative curvature and torsion relate to their classical counterparts through logarithmic–exponential transformations. Beyond the geometric construction, we propose a physical reading of the resulting invariants, relating the multiplicative curvature and torsion of the core curve, together with the tube radius, to field intensity and flux-conservation properties. This offers a unified perspective in which the geometry of the tube encodes physical characteristics of the underlying magnetic field.

Keywords: Magnetic flows, Quaternion algebra, Generalized tubes.
Mathematics Subject Classification (2020): 53A04, 53A45.

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INTERNATIONAL CONGRESS ON
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Pythagorean Hodograph Curves in Hyperbolic Space

Aziz Yazla

Selcuk University, azizyazla@gmail.com

Abstract. Pythagorean hodograph (PH) curves constitute an important class of parametric curves in computer-aided geometric design owing to their remarkable geometric and computational properties. While the classical theory of PH curves has been extensively developed in Euclidean geometry, analogous constructions in hyperbolic spaces have received comparatively little attention. Motivated by this, the present paper introduces and studies rational hyperbolic Pythagorean hodograph (H-PH) curves in the Poincaré upper half-space model of three-dimensional hyperbolic space.

A rational curve $\alpha(t) = (x(t), y(t), z(t))$ is defined to be an H-PH curve if its hodograph satisfies

$$[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2 = \sigma^2(t)z^2(t)$$

where $\sigma(t)$ is a rational function. Under the Poincaré metric, this condition is precisely equivalent to the rationality of the hyperbolic speed, thereby providing a natural extension of the classical Euclidean PH property to hyperbolic geometry. A characterization of rational H-PH curves is established by means of a rational quaternion representation of their hodographs, yielding necessary and sufficient conditions for the H-PH property. In particular, a characterization theorem for unit hyperbolic-speed H-PH curves is obtained by reducing the problem to a quaternion compatibility equation. The polynomial subclass is also investigated and shown to satisfy considerably stronger algebraic compatibility conditions than the rational class, revealing a fundamental distinction between polynomial and rational H-PH curves.

Keywords: Pythagorean hodograph curve, Poincaré upper half-space, curve characterization.
Mathematics Subject Classification (2020): 14H50, 53A35.

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Structural Characterization of α-Bronze Metric Manifolds

Neslihan Yıldırım[1], Mustafa Özkan [2]

[1] *Department of Mathematics, Graduate School of Natural and Applied Sciences, Gazi University, Ankara, Türkiye*, neslihanyildirim@gazi.edu.tr

[2] *Department of Mathematics, Faculty of Science, Gazi University, Ankara, Türkiye*, ozkanm@gazi.edu.tr

Abstract. In this study, we introduce α-bronze structures as a generalized framework encompassing both bronze and complex bronze structures. Using the bijective correspondence between α-metric and α-bronze metric manifolds, we transfer structural properties and obtain a complete classification of α-bronze metric manifolds in terms of the existence of the first canonical connection.

Keywords: Almost bronze structure, α-bronze structure, α-bronze metric manifold, α-metric manifold, Classification.
Mathematics Subject Classification (2020): 53C15, 53C05, 53C07.

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A Hyper-Dual Matrix Approach to Euclidean and Lorentzian Kinematics

Gülsüm Yüca[1], Yusuf Yaylı[2]

[1] *Faculty of Science, Department of Mathematics, Aksaray University, 68100, Aksaray, Turkey,*
gulsumbicer@gmail.com

[2] *Faculty of Science, Department of Mathematics, Ankara University, 06100, Ankara, Turkey,*
yayli@science.ankara.edu.tr

Abstract. The dual transformation between Euclidean and Lorentzian rotational matrices introduced by Dohi et al., this study extends this concept to the hyper-dual domain. We define a new transformation between hyper-dual orthogonal matrices in 3D Euclidean space and hyper-dual semi-orthogonal matrices in 3D Lorentzian space. Additionally, we demonstrate that the axes of rotation, representing dual lines in both geometric spaces, remain strictly invariant under the proposed transformation. Consequently, these results generalize existing dual transformations and offer a broader mathematical perspective for distinct geometric spaces.

Keywords: Hyper-dual matrices, Dual transformation, Kinematics.
Mathematics Subject Classification (2020): 53A35, 70E17, 70B10.

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Geometric Characterization of Ricci Solitons on Poly-Norden Space Forms

Semra Zeren[1], Feyza Esra Erdoğan[2]

[1] *Department of Engineering Fundamental Sciences, Faculty of Engineering and Natural Sciences, Sivas University of Science and Technology, Sivas, Türkiye*, zerensemra@sivas.edu.tr

[2] *Department of Mathematics, Faculty of Science, Ege University, İzmir, Türkiye*, feyza.esra.erdogan@ege.edu.tr

Abstract. Poly-Norden manifolds constitute an important class of geometric structures obtained by combining polynomial structures with Norden geometry and have attracted considerable attention in recent years. In this study, Ricci solitons and poly-Norden Ricci solitons on locally decomposable poly-Norden space forms are investigated. First, explicit expressions for the Ricci tensor, scalar curvature, poly-Norden Ricci tensor, and poly-Norden scalar curvature are derived from the curvature tensor of poly-Norden space forms. Subsequently, the Ricci soliton and poly-Norden Ricci soliton equations are examined, and new relations for the soliton constant are obtained, revealing its dependence on the trace of the structure tensor. The classifications of solitons are then established for Killing, homothetic, concurrent, and conformal Killing potential vector fields, and necessary and sufficient conditions characterizing shrinking, steady, and expanding solitons are determined. Furthermore, by employing the eigenvalue decomposition of the structure tensor, the influence of the poly-Norden structure on the soliton equations is clarified, and the fundamental geometric differences between Ricci solitons and poly-Norden Ricci solitons are highlighted. The obtained results extend previous studies on Ricci solitons associated with golden and Norden-golden structures to the broader framework of poly-Norden geometry and provide new insights into the relationship between poly-Norden structures and Einstein-type geometries.

Keywords: Poly-Norden manifold, Ricci soliton, Poly-Norden Ricci soliton, Einstein manifold, conformal Killing vector field.
Mathematics Subject Classification (2020): 53C15, 53C25, 53E20.

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